The QCD trace anomaly with 2+1 flavors of Highly Improved Staggered Quarks

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Lattice QCD and numerical setup

HISQ/tree – scale setting

Trace anomaly at low and high temperature

Conclusion

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Lattice QCD

- Quantum field theory on a discrete (Euclidean) space-time lattice, $(aN_s)^3 \times (aN_\tau)$, $T = 1/(aN_\tau)$, momentum cutoff π/a
- Evaluate path integrals stochastically (importance sampling)

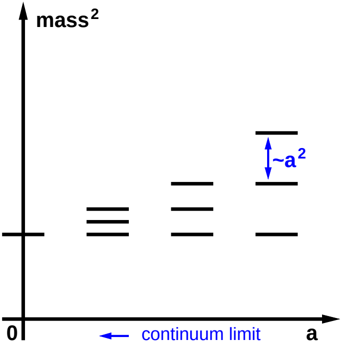
$$
\begin{aligned}
Z &= \int DUD\bar{\psi}D\psi \exp\{-S\}, \quad S = S_g + S_f \\
\langle \mathcal{O} \rangle &= \frac{1}{Z} \int DUD\bar{\psi}D\psi \mathcal{O} \exp\{-S\}
\end{aligned}
$$

- We use Highly Improved Staggered Quarks (HISQ)[1] and the tree-level Symanzik-improved gauge action, hence HISQ/tree
- The physics is recovered in the continuum limit ($a \to 0$, or $N_\tau \to \infty$ in the finite-temperature geometry)
- The trace anomaly

$$
\varepsilon - 3p = -\frac{T}{V}\frac{d \ln Z}{d \ln a} \quad \Rightarrow \quad \frac{p}{T^4} - \frac{p}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}
$$

[1] Follana et al. (2007), Bazavov et al. (2010)

Cutoff effects in the fermion sector



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- Staggered fermions lead to non-degenerate multiplets of states at $a > 0$ (e.g. pions) with the mass splitting $O(a^2)$
- On average this leads to a heavier than physical spectrum
- Worse for lighter states
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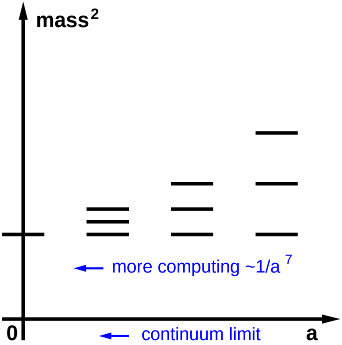

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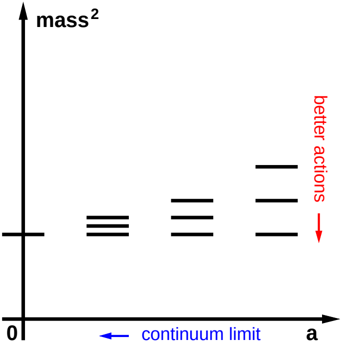

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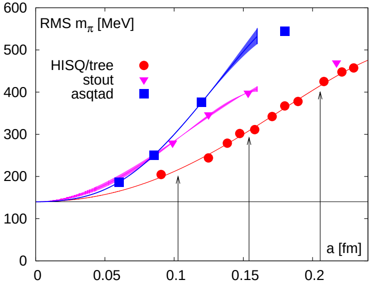
- "Improve" = modify the action with higher-dimensional operators that it has the same continuum limit and milder cutoff dependence

Cutoff effects in the fermion sector



- The root-mean-squared (RMS) pion mass for asqtad, stout and HISQ:

$$m_\pi^{RMS} = \sqrt{\frac{1}{16}\left(m^2_{\gamma_5} + m^2_{\gamma_0\gamma_5} + 3m^2_{\gamma_i\gamma_5} + 3m^2_{\gamma_i\gamma_j} + 3m^2_{\gamma_i\gamma_0} + 3m^2_{\gamma_i} + m^2_{\gamma_0} + m^2_{1}\right)}$$

- Vertical arrows indicate $T = 160$ MeV on lattices with $N_\tau = 6$, 8 and 12 (from right to left)

HISQ action

- Highly – More improvement compared to previously used staggered actions (e.g., asqtad)
- Improved – Adding higher-order (irrelevant) operators to the action allows for suppression of the discretization effects at $O(a^2)$
- Staggered – A particular fermion discretization scheme which partially deals with the fermion doubling problem, conserves a part of the chiral symmetry on the lattice and is relatively cheap to simulate numerically
- Quarks

HISQ/tree – numerical setup

- Calculation of the trace anomaly requires subtraction of UV divergences (take difference of zero- and finite-temperature quantities evaluated at the same values of the gauge coupling):

$$\frac{\varepsilon - 3p}{T^4} = R_\beta[\langle S_g\rangle_0 - \langle S_g\rangle_T] - R_\beta R_m[2m_l(\langle\bar{l}l\rangle_0 - \langle\bar{l}l\rangle_T) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)]$$

$$R_\beta(\beta) = -a\frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m}\frac{dm}{d\beta}, \quad \beta = \frac{10}{g^2}$$

- Line of constant physics $m_l = m_s/20$ (physical $m_l = m_s/27$)
- Statistics (in molecular dynamics time units):

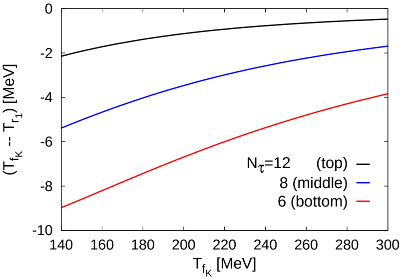
$T > 0$		$T = 0$	
$16^3 \times 4$	4K	$24^3 \times 32$	3-4K
$24^3 \times 6$	30K	32^4, $32^3 \times 64$	2-5K
$32^3 \times 8$	30K	48^4	8K
$40^3 \times 10$	10-20K	$48^3 \times 64$	8K
$48^3 \times 12$	30-40K	64^4	2-8K

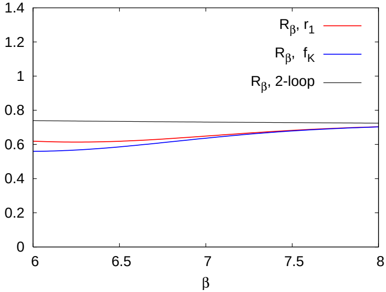
HISQ/tree - scale setting

- Sommer scale, $r_1 = 0.31$ fm (derived requiring that f_π is at the experimental value)

$$r^2 \frac{dV_{\bar{q}q}}{dr}\bigg|_{r=r_1} = 1$$

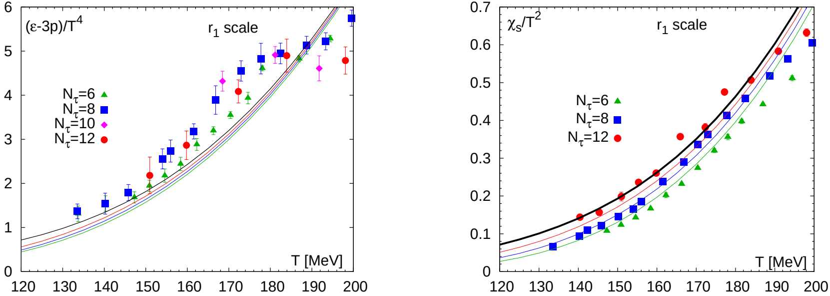
- Direct hadronic scale, f_K





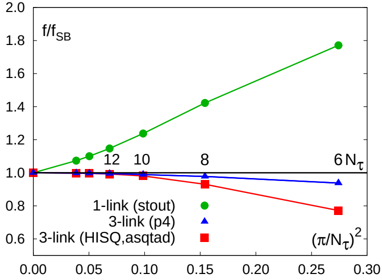
- Left: difference in temperature with lattice spacing set using r_1 or f_K
- Right: the β-function in r_1 and f_K scheme in the region of the inverse gauge coupling $10/g^2$ used in simulations

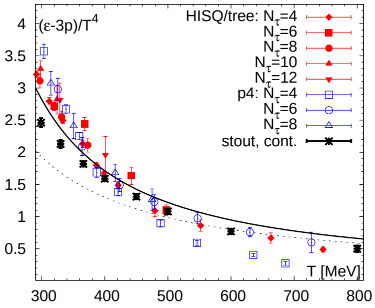
Trace anomaly at low temperature



- Left: the trace anomaly with the HISQ/tree action at $N_\tau = 6$, 8, 10 and 12
- Lines indicate the Hadron Resonance Gas result with the corresponding pion masses, mild cutoff dependence
- Right: the strangeness fluctuations are given for comparison, note more substantial cutoff dependence

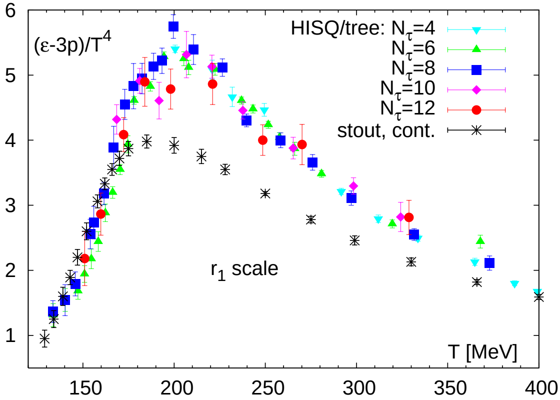
Trace anomaly at high temperature





- The free energy of an ideal quark gas at different N_τ (left)
- The trace anomaly at high temperature (right)
- Reasonable agreement with resummed perturbative calculations (solid line) using 2-loop running coupling. Dashed line is the 1-loop result.

Trace anomaly in the peak region



- Noticeable cut-off dependence in the peak region
- Need to increase statistics on $N_\tau = 10, 12$ ensembles before a continuum extrapolation may be attempted

Conclusion

- The HISQ/tree discretization scheme has better taste symmetry compared to other staggered actions (asqtad and stout). This improves the scaling behavior at low temperature.
- The HISQ/tree action also includes correction to the quark dispersion relation which is relevant at high temperatures.
- The scale setting procedure can make a difference at finite a, however, for HISQ/tree data at $N_\tau = 10$ and 12 the effect is negligible.
- The low-temperature behavior of the trace anomaly qualitatively follows the Hadron Resonance Gas model, but disagrees quantitatively.
- At high temperature the cutoff dependence expected from the free theory is not observed in the available range.
- Need higher statistics to control the continuum limit.