# Effective Gluon Potential and Yang-Mills Thermodynamics

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**based on:**
C.S. and K. Redlich, Phys. Rev. D **86**, 014007 (2012).

## "Confinement" in PNJL/PQM models

- NJL/QM under a constant background $A_0$

[Meisinger-Ogilvie (96), Fukushima (03), Ratti-Thaler-Weise (06)]

$$\mathcal{L}_{\rm kin} = \bar{q}\,(i\partial\!\!\!/ - A\!\!\!/_0)\,q$$
$$\Rightarrow\ \Omega_q = d_q\,T\int\frac{d^3p}{(2\pi)^3}\ln\left[1+3\Phi e^{-E/T}+3\Phi e^{-2E/T}+e^{-3E/T}\right]$$

$\langle\Phi\rangle\simeq 0$ at low T: 1- and 2-quark states thermodynamically irrelevant
$\Rightarrow$ mimicking confinement

- **pure gauge sector** of PNJL/PQM: $\Omega_g = T^4\,\mathcal{U}(\Phi;T)$
  - made based on $Z(3)$ symmetry:

$$\mathcal{U} = a(T)\bar{\Phi}\Phi + b(T)\left(\bar{\Phi}^3+\Phi^3\right) + c(T)\left(\bar{\Phi}\Phi\right)^2 + \cdots$$

  - T-dep. of coefficients $\Leftrightarrow$ Lattice EoS **"bottom-up"**
  - Polyakov-loop susceptibility from LQCD [Karsch-Laermann (94), Allton et al. (02)]
    $\Rightarrow$ insufficient $\mathcal{U}$ for fluctuations [CS-Friman-Redlich (06)]
  - where T-dep. comes from? $\cdots$ thermal gluon excitations $\Leftrightarrow$ $\mathcal{L}_{\rm YM}$

- revisit pure SU(3) YM theory
  closer contact with the underlying theory **"top-down"**
  $\Rightarrow$ better low-energy effective theory, thus better thermodynamics

**Q. How can gluons and $\Phi$ live together?**
**Gluon thermodynamics in low-T phase?**
**Similar mechanism to quark sector?**

# Deriving partition function from YM Lagrangian

- background field method, a constant uniform background $\bar{A}_0$

$$A_\mu = \bar{A}_\mu + g\check{A}_\mu$$

$$\bar{A}^a_\mu = \bar{A}^a_0 \delta_{\mu 0}\,, \quad \bar{A}_0 = \bar{A}^3_0 T^3 + \bar{A}^8_0 T^8$$

$$\sum_n \ln\det\left(D^{-1}\right) = \ln\det\left(1 - \hat{L}_A e^{-|\vec{p}|/T}\right)$$

[Gross-Pisarski-Yaffe (81)]

- Polyakov loop matrix in adjoint representation (8x8 matrix)

$$\hat{L}_A = \text{diag}\left(1\,,1\,,e^{i(\phi_1-\phi_2)}\,,e^{-i(\phi_1-\phi_2)}\,,e^{i(2\phi_1+\phi_2)}\,,e^{-i(2\phi_1+\phi_2)}\,,e^{i(\phi_1+2\phi_2)}\,,e^{-i(\phi_1+2\phi_2)}\right)$$

rank of SU(3) group $= 2 \;\Rightarrow$ elements expressed in 2 variables

- express in terms of

$$\Phi = \text{tr}\hat{L}_F/3\,,\ \bar{\Phi} = \text{tr}\hat{L}^\dagger_F/3$$

- full thermodynamics potential:

$$\Omega = \underbrace{\Omega_g}_{\sim a(T)\bar{\Phi}\Phi?} + \underbrace{\Omega_{\text{Haar}}}_{\text{responsible for Z(3) breaking}}$$

- full thermodynamics potential: $\Omega = \Omega_g + \Omega_{\mathrm{Haar}}$

$$\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \ln \left( 1 + \sum_{n=1}^{8} C_n(\Phi, \bar{\Phi})\, e^{-n|\vec{p}|/T} \right) ,$$

$$\Omega_{\mathrm{Haar}} = -a_0 T \ln \left[ 1 - 6\bar{\Phi}\Phi + 4 \left( \Phi^3 + \bar{\Phi}^3 \right) - 3 \left( \bar{\Phi}\Phi \right)^2 \right] ,$$

$$C_1 = C_7 = 1 - 9\bar{\Phi}\Phi , \quad C_2 = C_6 = 1 - 27\bar{\Phi}\Phi + 27 \left( \bar{\Phi}^3 + \Phi^3 \right) ,$$

$$C_3 = C_5 = -2 + 27\bar{\Phi}\Phi - 81 \left( \bar{\Phi}\Phi \right)^2 ,$$

$$C_4 = 2 \left[ -1 + 9\bar{\Phi}\Phi - 27 \left( \bar{\Phi}^3 + \Phi^3 \right) + 81 \left( \bar{\Phi}\Phi \right)^2 \right] , \quad C_8 = 1$$

$\Rightarrow$ energy distributions solely determined by group characters of SU(3)

– no free parameter in $\Omega_g$

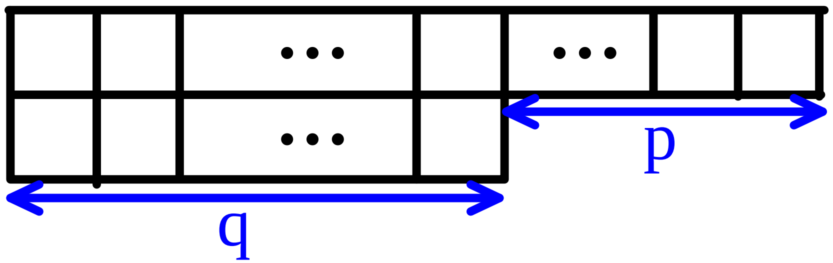
– one parameter in $\Omega_{\mathrm{Haar}}$: $a_0 \Leftrightarrow T_c^{\mathrm{lat}} = 270$ MeV

## Character expansion of $\Omega_g$

- effective action in the strong coupling exp. [Wozar-Kaestner-Wipf-Heinzl-Pozsgay (06)]

$$\mathcal{S}_{\text{eff}}^{(\text{SC})} = \lambda_{10} S_{10} + \lambda_{20} S_{20} + \lambda_{11} S_{11} + \lambda_{21} S_{21}$$

$S_{pq}$: products of SU(3) characters $\sim$ a series of $Z(3)$-inv. operators




$$C_{1,7} = S_{10}\,, \quad C_{2,6} = S_{21}\,,$$
$$C_{3,5} = S_{11}\,, \quad C_4 = S_{20}$$

- a "minimal" model: $\mathcal{S}_{\text{eff}} = \lambda S_{10} \sim \lambda \bar{\Phi}\Phi$ plus $\mathcal{S}_{\text{Haar}}$
  $\Rightarrow$ 1st-order phase transition

- coefficient $\lambda$ can be deduced from $\Omega_g$! $\quad \Omega_g \simeq \mathcal{F}(T)\bar{\Phi}\Phi$

- cf. "phenomenological" potentials used in PNJL/PQM
  $\Omega = a(T)T^4\bar{\Phi}\Phi + \Omega_{\text{Haar}}$: unknown $a(T)$ fixed by fitting Lattice EoS

## Thermodynamics

- high temperature limit: $\Phi \to 1 \Rightarrow$ non-int. gluon gas

$$\Omega_g(\Phi = \bar{\Phi} = 1) = 16T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - e^{-|\vec{p}|/T}\right)$$

- any finite temperature in confined phase: $\Phi = 0$ thus $\Omega_{\mathrm{Haar}} = 0$

$$\Omega_g(\Phi = \bar{\Phi} = 0) \sim 2T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-|\vec{p}|/T}\right)$$

**wrong sign!** $\Rightarrow$ unphysical EoS $s, \epsilon < 0$

**Gluons are NOT correct dynamical variables below $T_c$!**

cf. PNJL/PQM: quarks are suppressed but exist at any T.

- **higher representations of Polyakov loop**
  - non-vanishing in confined phase *within mean field approx.*
  - do not condense when energy distributions are expressed in fund. rep.

$\Rightarrow$ the correct physics restored!

# A hybrid approach for Yang-Mills thermodynamics

- below $T_c$: no gluons but **glueballs**
  $\Rightarrow$ introduce scalar glueballs as dilatons $\chi$
- **QCD trace anomaly**:
  $T_\mu^\mu \sim \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle$ scale symmetry breaking due to gluon condensate
  $\Rightarrow$ this is encoded with dilaton potential such that $T_\mu^\mu \sim \chi^4$ [Schechter (80)]

$$V_\chi = \frac{B}{4} \left( \frac{\chi}{\chi_0} \right)^4 \left[ \ln \left( \frac{\chi}{\chi_0} \right)^4 - 1 \right]$$

- thermodynamics of glueballs

$$\Omega = \Omega_\chi + V_\chi + \frac{B}{4},$$

$$\Omega_\chi = T \int \frac{d^3p}{(2\pi)^3} \ln \left( 1 - e^{-E_\chi / T} \right),$$

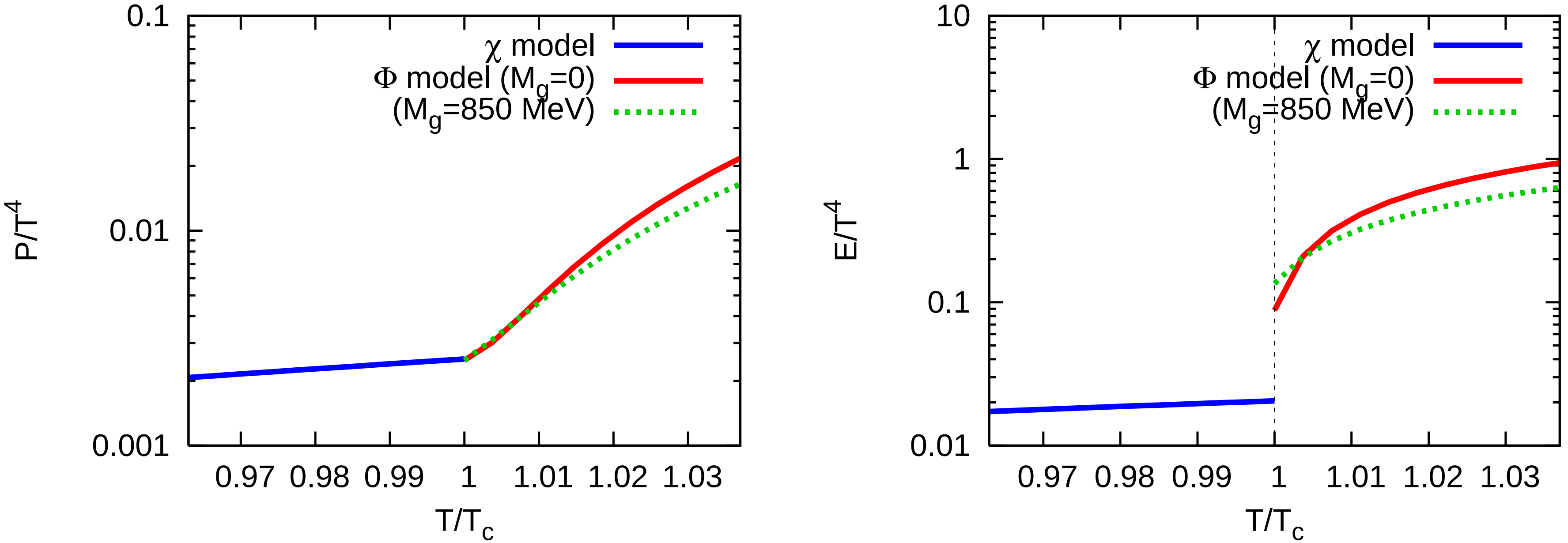
$$E_\chi = \sqrt{|\vec{p}|^2 + M_\chi^2}, \quad M_\chi^2 = \frac{\partial^2 V_\chi}{\partial \chi^2},$$

- below $T_c$: no gluons but glueballs
  $\Rightarrow$ switching dynamical variables at $T_c$

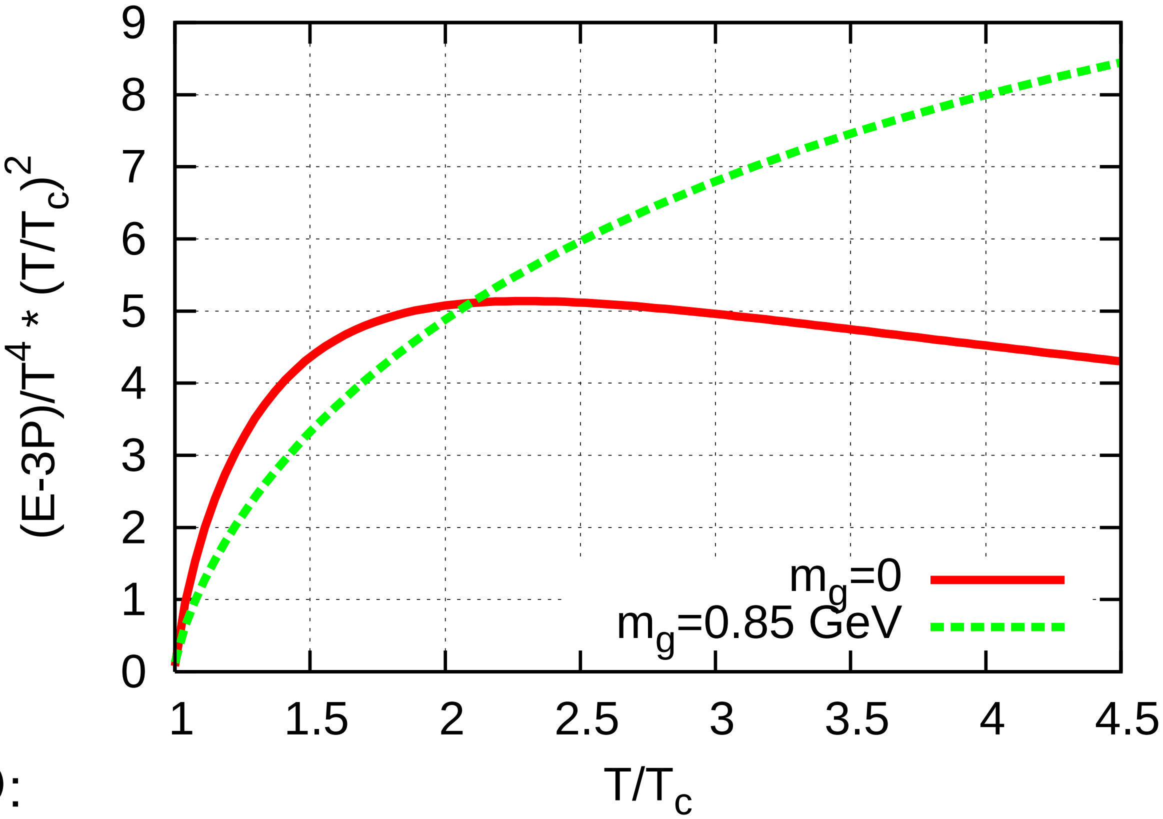
$$\Omega = \Theta(T_c - T)\,\Omega(\chi) + \Theta(T - T_c)\,\Omega(\Phi)\,,$$
$$\Omega(\chi) = \Omega_\chi + V_\chi + B/4\,, \quad \Omega(\Phi) = \Omega_g + \Omega_{\rm Haar} + c_0$$

- $\epsilon_0 = 0.6$ GeV fm$^{-3}$, $M_G = 1.7$ GeV, $T_c = 0.27$ GeV, $P(\chi; T_c) = P(\Phi; T_c)$

$$B = (0.368\,\text{GeV})^4\,, \quad \chi_0 = 0.16\,\text{GeV}$$
$$a_0 = (0.197\,\text{GeV})^3\,, \quad c_0 = -(0.180\,\text{GeV})^4$$

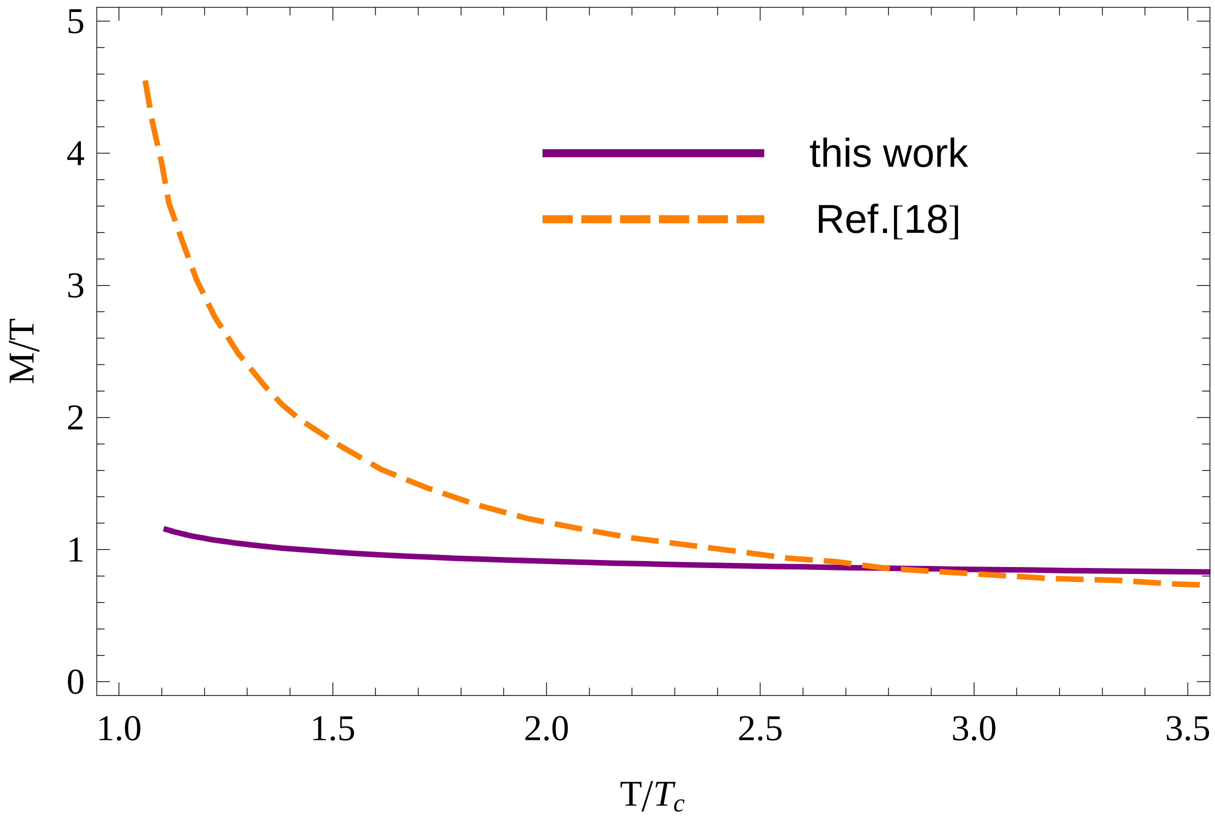


## Interaction measure: massive gluon?




- Lattice QCD:
  - $I(T)/T^4 \sim A/T^2 + B/T^3 + C/T^4$ [Borsanyi et al. (12)]
  - $T/T_c < 3\text{-}4$: $I/T^2 \sim$ const. due to "residual interaction"
- model: $E_g^2 = p^2 + M_g^2$
  - massless gluons: $I \sim T^2$, high T?
  - massive gluons: $I \sim T^3$, condensate of dilaton? resummed $g(T)T$?

- effective gluon mass extracted from lattice EoS [Ruggieri et al. (12)]




  – based on quasi-gluon models w/ Polyakov loops vs. w/o Polyakov loops
  – role of Polyakov loop: $M/T \sim \mathcal{O}(1)$!
  – cf. QPM w/o Polyakov loop: $M \gg T_c$

- standard dispersion $E_g = \sqrt{p^2 + M_g^2}$? or modified in hot medium?

  $\Rightarrow$ non-trivial p-dep. would come in. cf. pQCD

## How to introduce quarks?

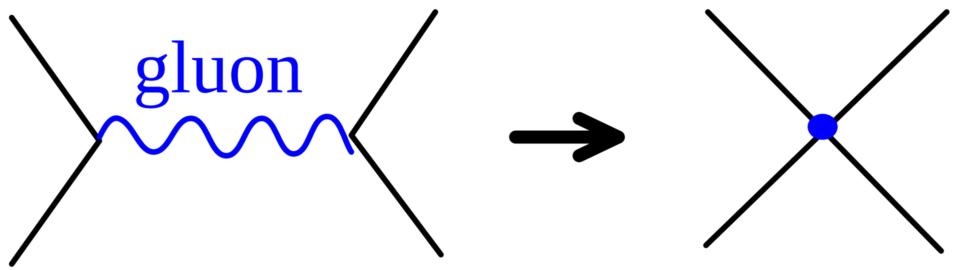
- pert. part $(T < T_c, \Phi \sim 0)$

$$\Omega_{g+q} \sim 2T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-E_g/T}\right) - 4N_f T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-3E_q/T}\right)$$
$$\sim \frac{T^2}{\pi^2} \left[ M_g^2 K_2\left(\frac{M_g}{T}\right) - \frac{2N_f}{3} K_2\left(\frac{3M_q}{T}\right) \right]$$

with effective masses: $M_g \equiv M_{\rm gluball}/2\,, \quad M_q \equiv M_{\rm nucleon}/3$

therefore $s = -\partial\Omega/\partial T \overset{?}{\sim} -\partial\Omega_{g+q}/\partial T < 0$

- non-pert. part $\sim$ **effective 4-fermi interaction**




$\cdots$ non-local interaction: $T, \Phi, p$ $\Rightarrow$ crossover at $\mu = 0$

## Summary

- **role of Polyakov loops in quasi-particle approaches**
- **derivation of gluon partition function from YM Lagrangian**
  - Polyakov loops naturally appear representing group character.
  - gluons are forbidden below $T_c$ dynamically.
  - a hybrid approach.
- **higher representations of Polyakov loop**
  - non-vanishing even in confined phase: mean field artifact
  - do not condense when energy distributions are expressed in fund. rep.

- **Polyakov-loop susceptibilities vs. LQCD** [Lo-Friman-Kaczmarek-Redlich-CS]
- **dynamical gluon mass** [CS-Mishustin-Redlich]
- **introducing quarks**