QCD Equation of State From a Chiral Hadronic Model Including Quark Degrees of Freedom

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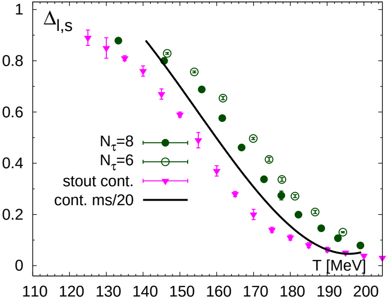
Outline

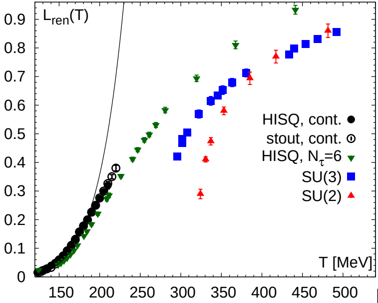
Phase Transitions – Order Parameter from Lattice QCD

Chiral Condensate



[1]

Polyakov Loop



[2]

- smooth cross over in chiral and deconfinement transition
- transition temperatures converge in newest continuum extrapolated lattice data

Chiral

$T_c \approx 160$ MeV (in hadronic region)

Deconfinement

$T_c \approx 200$ MeV

[1] A. Bazavov *et al.* [HotQCD], Phys. Part. Nucl. Lett. **8**, 860 (2011), [2] P. Petreczky, arXiv:1301.6188.

Chiral Effective Model

Chiral hadronic model based on a **nonlinear realization of an σ – ω model.**
Interactions mediated by scalar and vector meson fields.
Degrees of freedom in hadronic sector:

- scalar, pseudoscalar, and vector mesons,
- baryonic octet and decuplet,
- all hadronic resonances up to $m = 2.6$ GeV,
- gluon condensate (dilaton field χ).

Terms in Effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_{\mathrm{BS}} + \mathcal{L}_{\mathrm{BV}} + \mathcal{L}_{\mathrm{V}} + \mathcal{L}_{\mathrm{S}} + \mathcal{L}_{\mathrm{SB}}$$

GC Potential

$$\Omega/V = -\mathcal{L} - \Omega_{\mathrm{th}}$$

- minimize potential with respect to fields $\Rightarrow$ equations of motion
- self-consistent equations $\Rightarrow$ field values, densities & thermodyn. quantities

P. Papazoglou, S. Schramm, J. Schaffner-B., H. Stöcker, W. Greiner, Phys. Rev. C **57**, 2576 (1998).
P. Papazoglou, D. Zschiesche et al., Phys. Rev. C **59**, 411 (1999).

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- Parameters fixed by hadronic vacuum observables, nuclear matter saturation properties, and symmetry relations,.
- Model provides satisfactory description of **finite nuclei** and **neutron stars**.

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P. Papazoglou, D. Zschiesche et al., Phys. Rev. C **59**, 411 (1999).

Polyakov Loop Potential & Quarks

- introduce effective potential as in PNJL Model [3], with parameters chosen such as to fit pure gauge results for $\Phi(T)$

$$U = -\frac{1}{2}\, a(T)\bar{\Phi}\Phi + b\left(\frac{T_0}{T}\right)^3 \ln\left[1 - 6\bar{\Phi}\Phi \; + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2\right]$$

- couple quarks to Polyakov loop Φ in thermal energy

$$\Omega_{\mathrm{q\bar{q}}} = -T \sum_{i \in q} \frac{\gamma_i}{(2\pi)^3} \int d^3k \; \left(\ln\left[1 + \Phi\, e^{-\frac{1}{T}\left(E_i^*(k) - \mu_i^*\right)}\right] + \ln\left[1 + \bar{\Phi}\, e^{-\frac{1}{T}\left(E_i^*(k) + \mu_i^*\right)}\right] \right)$$

- include thermal energy of quarks in model $\Omega_{\mathrm{th}} = \Omega_{\mathrm{q\bar{q}}} + \Omega_{\mathrm{B\bar{B}}} + \Omega_{\mathrm{M}}$

- Quarks fully couple to scalar/vector meson fields with coupling parameters according to additive quark model.
- They do not populate the ground state.

[3] C. Ratti, S. Roessner, M. A. Thaler and W. Weise, Eur. Phys. J. C **49**, 213 (2007).

Interactions of Particles with Meson Fields

effective masses:

generated by scalar fields:
$m_i^* = g_{i\sigma}\sigma + g_{i\zeta}\zeta + \delta m_i$

effective potentials:

generated by vector fields:
$\mu_i^* = \mu_i - g_{i\omega}\omega - g_{i\phi}\phi$

Resonance couplings to vector fields: (via coefficient r_v)

vector couplings:

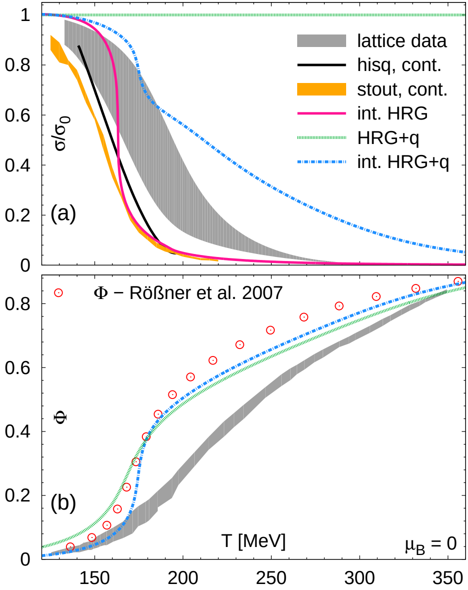
$g_{B\omega,\phi} = r_v \cdot g_{N\omega,\phi}$

- resonance scalar coupling fixed $r_s \sim 1$ ensuring smooth cross over at $\mu_B = 0$
- vector coupling has big impact on phase diagram and fluctuations at the phase transition.
- **Non-Interacting HRG:** set all couplings to zero & masses fixed at vacuum masses

Excluded Volume Effects: ($V_{\text{hadrons}} \neq 0$, $V_{\text{quarks}} = 0$)

- suppress hadronic d.o.f. at higher densities via eigenvolume of particles
- reduces pressure as a function of temperature
- e, ρ, s are volume-corrected in order to maintain thermodyn. consistency

Order Parameter σ/σ_0 and Φ at $\mu_B = 0$



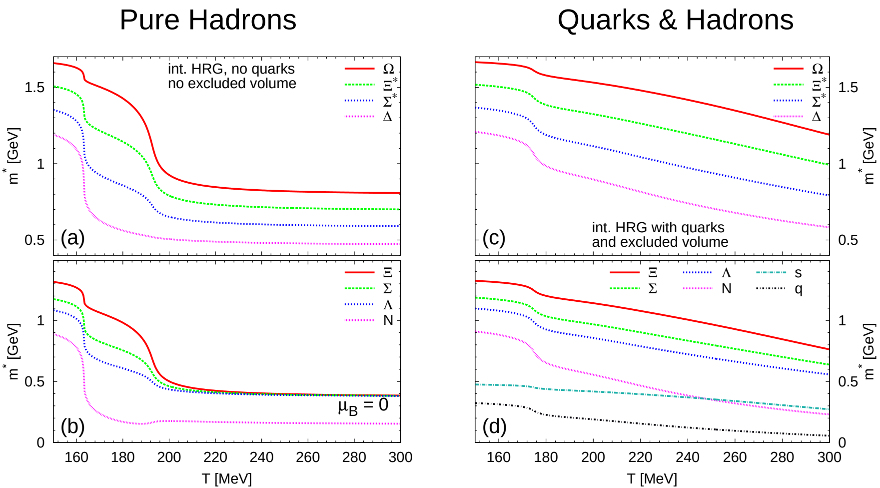
chiral:

- smooth continuous transition – cross over
- purely hadronic model: $T_C \approx 160$ MeV
- hadrons & quarks: $T_C \approx 175$ MeV
- slow decrease in quark phase due to ex. volume suppression

deconfinement:

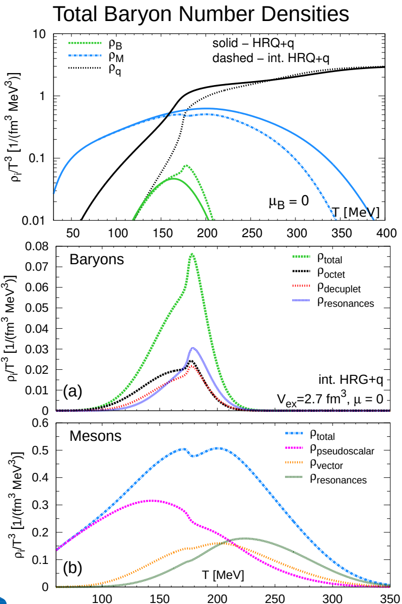
- transition follows PNJL curve, lattice much slower
- transition temperature as in σ with quark phase
- smoother curve without field interactions

Effective Masses m_i^* at $\mu_B = 0$



- $m_i^*(T)$ drop to explicit mass δm_i with higher T; scalar fields σ, ζ define slope
- purely hadronic: sharp drop in m^* at T_c
- hadrons & quarks: slower decrease in m^* over larger T-scale

Particle Densities in Transition Region



solid lines: no interactions with fields
dashed lines: fully interacting model

at T_c:

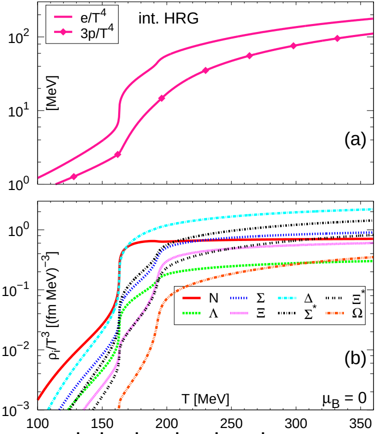
- sharpest rise of quark+antiquark density
- baryons & mesons suppressed due to $V_{\rm ex} \neq 0$
- quarks dominant above T_c
- pure quarks at $T \geq 350$ MeV
- baryon and meson resonance states have significant multiplicities at T_c and slightly above

$\Rightarrow$ Chiral Phase transition ($T_c = 160$ MeV) driven by hadron d.o.f. to a large extent

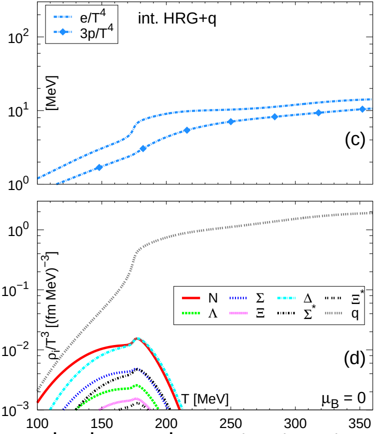
Heavy-mass resonances should not be neglected in HRG

Energy & Pressure – Contribution of Particles

Pure Hadrons

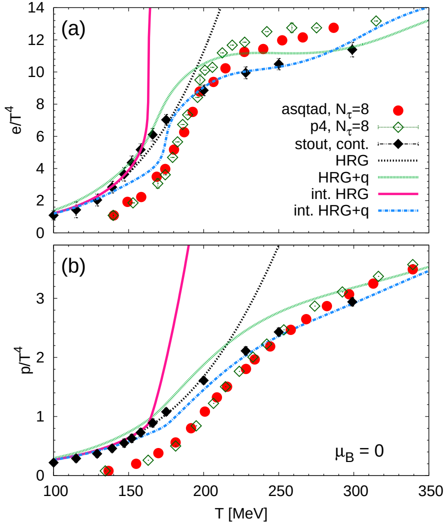


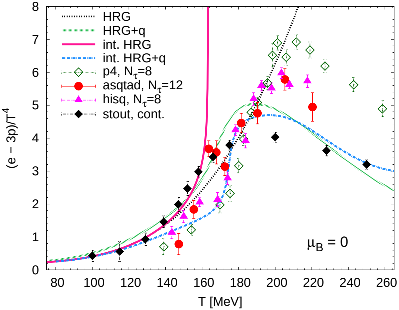
Quarks & Hadrons



- purely hadronic: drop in masses at T_c causes rise in e and p - saturates at larges values for high T
- hadrons & quarks: suppression of baryons causes much smaller e, p - only quark contribution at $T > 200$ MeV

Thermodynamic Quantities at $\mu_B = 0$ Compared to Lattice





Energy density, pressure, and interaction measure

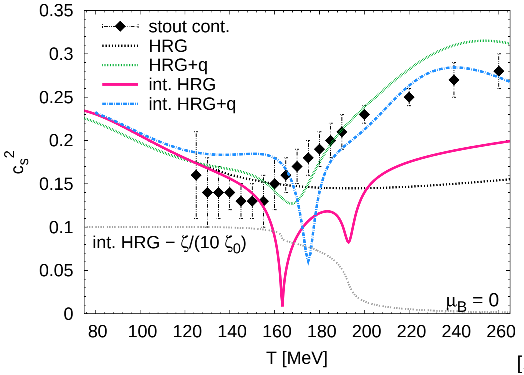
- best agreement between full interacting hadron+quark model and stout continuum extrapolated data

Speed of Sound

Collision dynamics (expansion of fireball) characterized by velocity of sound

sound velocity

$$c_s^2 = \left.\frac{\partial p}{\partial \varepsilon}\right|_{s/A}$$

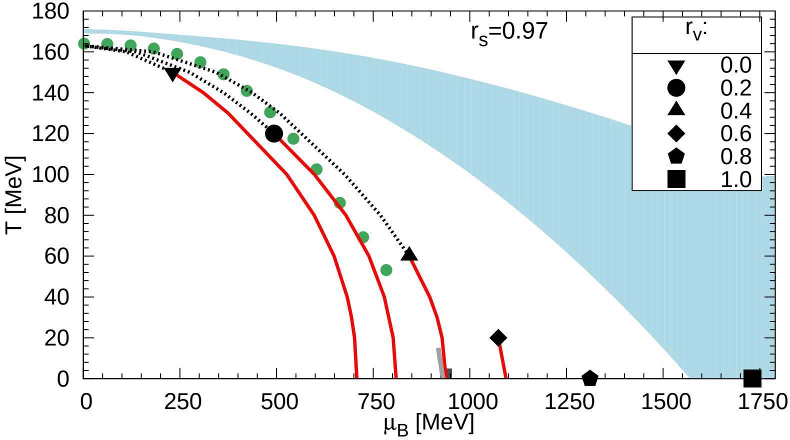


[1]

- all scenarios with phase transition show softening at T_c
- interacting model without quarks shows lowest c_s^2 (steepest rise in e and p at T_c)
- fully interacting models show significant more softening than lattice data [1]

[1] S. Borsanyi, G. Endrodi, Z. Fodor, et al., JHEP **1208**, 053 (2012)

Phase Diagram from Purely Hadronic Model

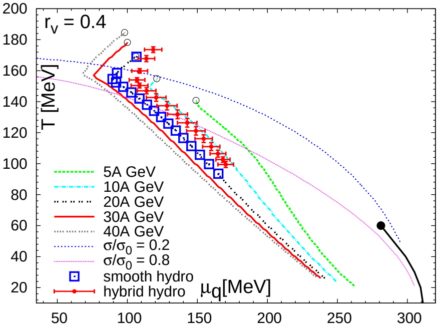


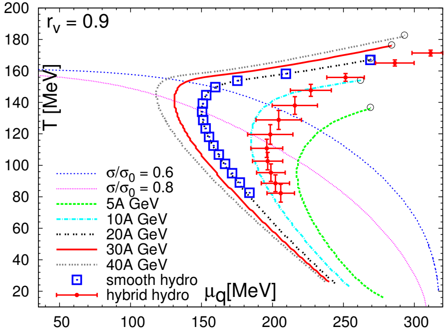
- resonance vector coupling has major impact on order and position of p.t.-lines and CEP.
- Chiral p.t. moves to higher potentials & CEP to lower temperatures with increasing vector coupling strength.
- no 1st order p.t. but only broad cross over for $r_v > 0.6$.

[1] C. R. Allton, S. Ejiri, et al., Phys. Rev. D **66**, 074507 (2002) [2] P. Braun-Munzinger, K. Redlich, J. Stachel[arXiv:nucl-th/0304013]

Dynamic Expansion Paths – Isentropes I – Pure Hadrons

resonance coupl. $r_v = 0.4$ $r_v = 0.9$





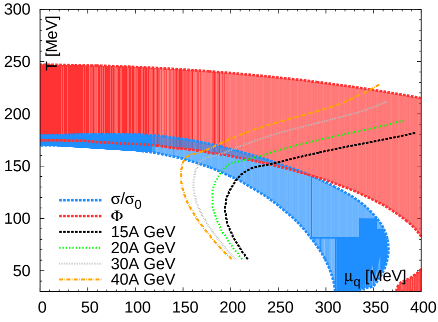
- lines of constant entropy per baryon (s/A), smooth hydro (blue squares) and hybrid transport (red crosses) at $E_{\rm lab} = 20$ AGeV
- slope of isentropes reflect smoother & broader transition in σ

geometric overlap model

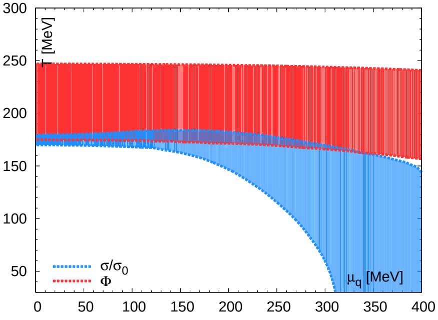
$$\rho_{\rm in} = 2\gamma_{c.m.}\rho_0, \quad \epsilon_{\rm in} = \sqrt{s}\gamma_{c.m.}\rho_0, \qquad \rho_0 = 0.15 \text{ fm}^3$$

Dyn. Expansion Paths – Isentropes II – Hadrons & Quarks

$g_{q\omega} = 0$



$g_{q\omega} = 1/3\, g_{N\omega} = 4.0$



- Transition regions: red – $\Phi = 0.4 \rightarrow 0.7$, blue $\sigma/\sigma_0 = 0.7 \rightarrow 0.4$. with increasing T.
- lack of repulsive quark vector coupling causes narrow transition region at larger $\mu \rightarrow$ presence of quarks in confined phase possible
- reasonable quark vector coupling $\Rightarrow$ chiral transition as in purely hadronic model

Quark Number Susceptibilities

Fluctuations of conserved charges sensitive indicators for the structure of a thermal medium produced in HIC.
Susceptibilities hint to net-baryon number fluctuations and therefore to a change in the underlying d.o.f. at phase transition

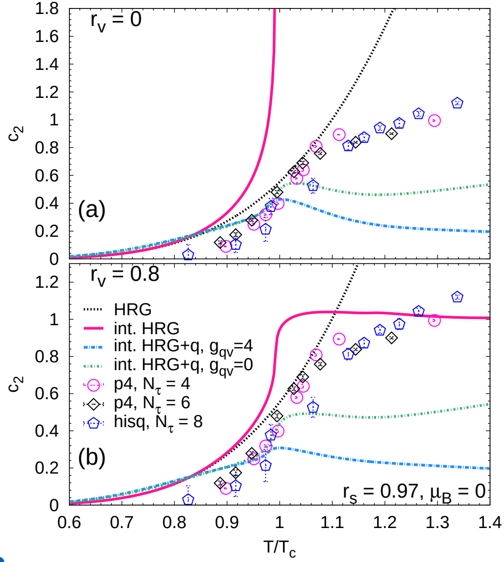
Taylor expand the pressure $p = -\Omega/V$ with respect to $\frac{\mu}{T}$

$$\frac{p(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_B}{T}\right)^n$$

quark number susceptibilities

$$c_n(T) = \frac{1}{n!} \frac{\partial^n (p(T,\mu_B)/T^4)}{\partial(\mu_B/T)^n}\bigg|_{\mu_B=0}$$

Quark Number Susceptibilities at $\mu_B = 0$ – c_2

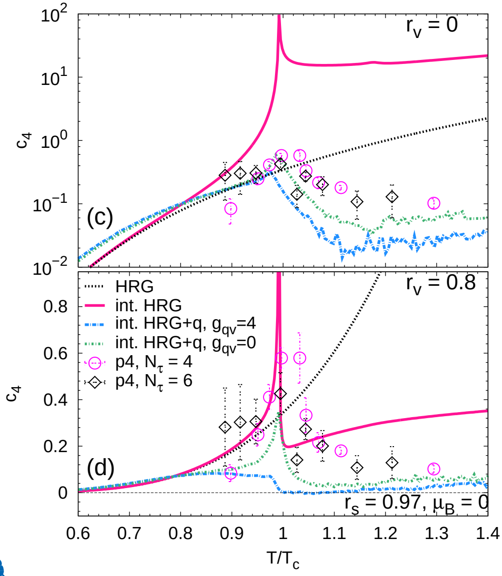


Second order susceptibilities c_2:

- Large overestimation without vector couplings $r_v = 0$.
- maximum fluctuations at T_c for $r_v = 0.8$
- fluctuations effectively suppressed by vector couplings of baryons and of quarks

$\Rightarrow$ repulsive vector field decreases fluctuations significantly.

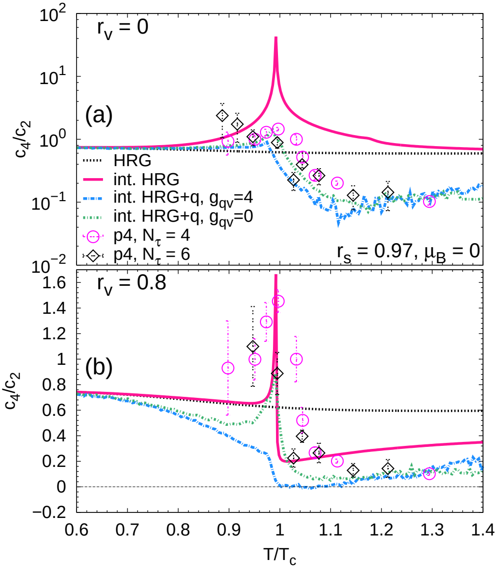
Quark Number Susceptibilities at $\mu_B = 0$ – c_4



Fourth order susceptibilities c_4:

- similar observations can be made:
- overestimation without vector interactions
- repulsive vector interactions decrease fluctuations significantly

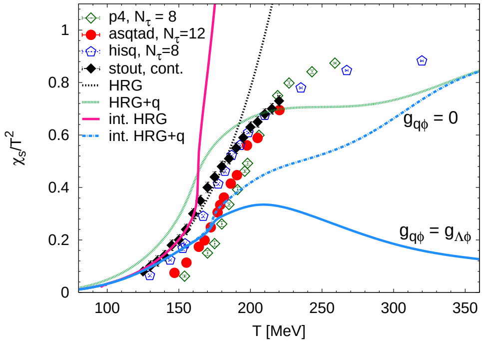
Susceptibility ratios at $\mu_B = 0$ – c_4/c_2



Susceptibility ratios c_4/c_2:

- ratios much easier to obtain experimentally
- same arguments apply for susceptibility ratios c_4/c_2
- significant discrepancies in fluctuations from lattice QCD and model

Strange Quark Number Susceptibility at $\mu_B = 0$



strange quark susceptibility

Analogous to non-strange quark potential:

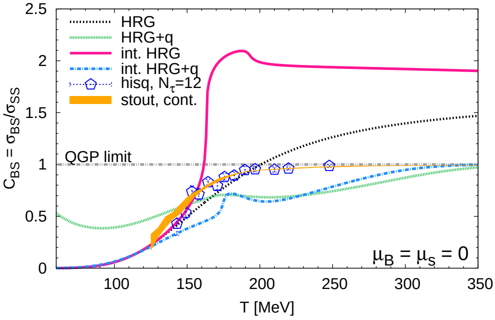
$$\chi_s = T^2 \left. \frac{\partial^2(p(T,\mu_s))}{\partial \mu_s^2} \right|_{\mu_s=0}$$

Strange quark susceptibility c_s:

- Good agreement between stout data and (int.) HRG up to $\sim T_c$
- lack of strange quark vector repulsion (green curve) causes higher c_s at $T < T_c$
- strange susceptibilities suppressed by strange quark vector coupling

$\Rightarrow$ **all susceptibilities (and the corresponding fluctuations) are highly sensitive to the respective vector couplings!**

Baryon Strangeness Correlator – c_{BS}



- excess of strange baryons at $T > T_c$ in hadronic scenario due to drop in effective masses
- slow transition for int. hadrons + quarks to QGP value caused by existence of strange mesons up to high T
- without vector repulsion: quarks appear below T_c
- kink in functions caused by baryon mass dropping and subsequent excluded volume suppression of strange baryons

BS Correlator

$$C_{BS} \approx -3 \frac{\sum_i \rho_i \, B_i S_i}{\sum_i \rho_i \, S_i^2},$$

Conclusions

- Model incorporates correct d.o.f. for a wide range in temperature and density.
- Model exhibits both, chiral and deconfinement phase transition $T_c \approx 170$ MeV (smooth cross over at $\mu_B = 0$).
- Phase transition driven by hadrons to a large extent
- Heavy-mass hadron resonances are important in the transition region
- Phase structure and location of phase transition is highly dependent of the particles' couplings to the repulsive vector fields.
- Thermodynamic properties and susceptibilities in qualitatively fair agreement with newer lattice QCD.
- Fluctuations & susceptibilities highly sensitive to vector interactions.