Charge fluctuations at QCD crossover and effect of volume fluctuations

Vladimir Skokov

BROOKHAVEN NATIONAL LABORATORY

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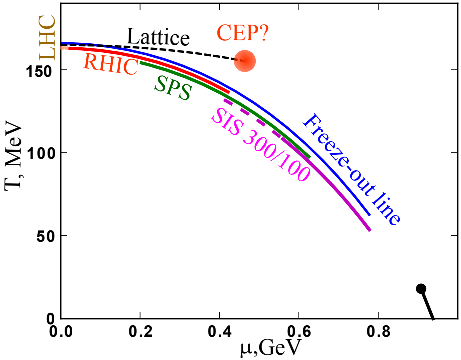
Outline

- Introduction and motivation
- Fluctuations of conserved charges at crossover
- Volume fluctuations
- Volume fluctuations in equilibrium chiral model
- Conclusions

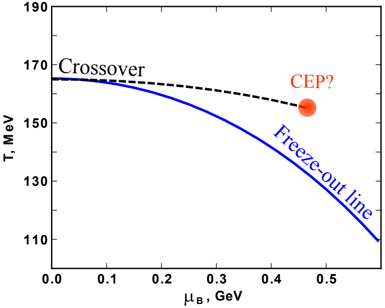


Structure of the phase diagram

- crossover at small μ_B

  Lattice QCD: results are consistent with underlying O(4) universality class
- (Expected) critical end point, 3d Ising model universality class
- (Expected) first-order transition
- (Expected) quarkyonic/inhomogenious phase

Curvature of transition line and freeze-out



Detailed analisys shows that crossover line and freeze-out approximately coincide even for higher chemical potentials (talk by S. Mukherjee)

- LQCD: curvature of crossover line μ_B

$$T/T_c \approx 1 - \mathbf{0.0066}\,(\mu_B/T)^2$$

- HRG model: freeze-out curve

$$T/T_c \approx 1 - \mathbf{0.023}(\mu_B/T)^2$$

Fluctuations of conserved charges

Experiment:

- Event-by-event fluctuations in given centrality class
- Cut phase space to minimize impact of conservation laws
  Nonetheless conservation laws are important to take into account
  A. Bzdak talk; A. Bzdak, V. Koch, V.S., Phys.Rev. C87 (2013) 014901
- $\mathcal{P}(N) \rightsquigarrow \langle (N-\bar{N})^k \rangle = \sum_N (N-\bar{N})^k P(N) \rightsquigarrow$ cumulants

Theory:

- Grand-canonical thermodynamics
- Conserve quantities are associated with corresponding Lagrange multipliers
- Derivatives with respect to appropriate Lagrange multipliers provide cumulants for fluctuations of conserved charges
- $p(T,\mu) \rightsquigarrow \partial^n/\partial\mu^n \, p(T,\mu) \rightsquigarrow \chi_n \cdot (VT^3) \equiv$ cumulants

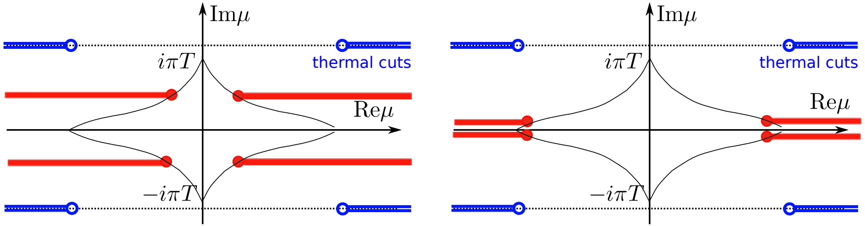
Hadron resonance gas models at $\mu_S = \mu_Q = 0$:

Baryon number fluctuations:

- $T << m_p \rightsquigarrow$ Boltzmann approximation: $p/T^4 = \sum_i f(m_i/T) \cosh(\mu_B/T) + g(T)$
- $\chi_{2n} \propto \cosh(\mu_B/T) \quad \chi_{2n+1} \propto \sinh(\mu_B/T)$
- $\chi_{2n}/\chi_2 = 1 \quad \chi_{2n+1}/\chi_1 = 1$
- Positivity $\chi_{2n} > 0$
- Effect of statistics $\chi_6/\chi_2 =$ 0.95 at $\sqrt{s} = 10$ GeV
- Electric charge fluctuations: ratios > 1 due to Bose statistics of pions and multiple charged hadrons

- Mathematically one expects, that higher order cumulants will be negative at $T = T_{pc}$, $\mu_B = 0$
- Crossover is characterized by singularities in complex μ_B plane

crossover phase transition: close to CP T->Tc:



see M. Stephanov PRD 73 094508 (2006); V.S.,K. Morita, B. Friman PRD 2010

- Darboux's theorem: some higher order cumulants (coefficients of Taylor expansion at $\mu_B = 0$) are negative

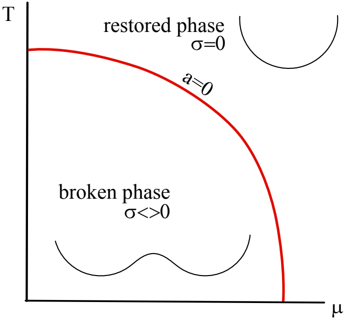
- Lattice QCD: light quark mass dependence is consistent with O(4) scaling and if so, QCD at physical pion mass in O(4) scaling regime
- Therefore, based on universality argument, we can use O(4) toy model to understand grand properties of cumulants

Landau theory for 2d-order phase transition ($m_\pi = 0$):

$$\Omega = \frac{a}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4$$

σ - order parameter

$a = \frac{1}{t_0}\left[\left(\frac{T}{T_c} - 1\right) + \kappa\,(\mu_B/T)^2\right]$



Minimization of Ω: $\partial\Omega/\partial\sigma=0$ leads to

$\sigma^2_{\rm min} = -\frac{a}{\lambda}$ for $a < 0$ and $\sigma^2_{\rm min} = 0$ for $a > 0$.

Pressure: $p = -\Omega(\sigma = \sigma_{\rm min}) = \frac{a^2}{4\lambda}$

Second-order cumulant $\mu = 0$: $\chi_2 \sim (T - T_c)\theta(T - T_c)$

Higher order cumulants $n > 4$ $\chi_n = 0$

Fluctuations of order parameter $\rightsquigarrow$ **non-trivial** critical exponents

Pressure: $p \sim a^2$ $\rightsquigarrow p \sim a^{2-\alpha}$

$a = \frac{1}{t_0}\left[\left(\frac{T}{T_c} - 1\right) + \kappa\,(\mu_B/T)^2\right]$

α is specific heat critical exponent $\alpha_{O(N)} < 0$ for $N \geq 2$

$\alpha_{O(4)} \approx -0.21$

$\mu = 0$: higher cumulants are non-trivial: $\chi_n \sim (T - T_c)^{-\frac{1}{2}(n-4+2\alpha)}$

$\chi_4 \sim (T - T_c)^{-\alpha}$ finite at T_c for O(4) (but divergent for Z(2))

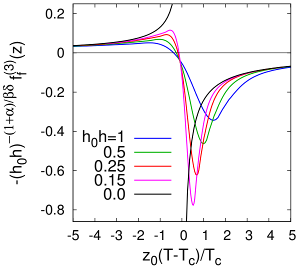
$\chi_6 \sim 1/(T - T_c)^{1+\alpha}$ divergent at T_c

Beyond mean-field: O(4) scaling functions on lattice

Based on: J. Engels, F. Karsch, arXiv:1105.0584 and
B. Friman et. al., arXiv:1103.3511

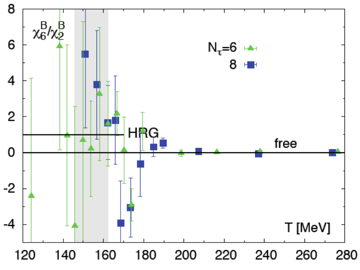
Lattice simulations of O(4) models
⤳ singular part of

$$p/T^4 \propto -f(a,h)/T^4, \quad h \propto m_q$$



$\chi_6(\mu = 0)$

Does singular part dominates in QCD?
No precision LQCD calculations of χ_6^B



C. Schmidt, EMMI Rapid reaction task force meeting 2013

talk by S. Mukherjee

Polyakov-loop extended quark-meson model

Polyakov loop-extended quark meson model

- O(4) symmetry
- quark interaction with Polyakov loops $\rightsquigarrow$ statistical confinement
- in O(4) scaling regime for physical pion mass

- Model based on symmetries of QCD
  $SU(2)_L \otimes SU(2)_R$, scalar condensate $\sigma \propto \langle \bar{q} q \rangle$
  $Z(3)$, Polyakov loop $\ell = \frac{1}{N_c} \langle \mathrm{Tr}_c \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\boldsymbol{x}, \tau) \right] \rangle$
- Lagrangian of PQM model:

$$\mathcal{L} = \bar{q} \left[i\gamma^\mu D_\mu - g(\sigma + i\gamma_5 \boldsymbol{\tau\pi}) \right] q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 - U(\sigma, \boldsymbol{\pi}) - \mathcal{U}(\ell, \ell^*)$$

$\mathcal{U}(\ell, \ell^*)$ – Z(3)-invariant Polyakov loop potential
Gluons are coupled to quarks q by covariant derivative
$D_\mu = \partial_\mu - iA_\mu$

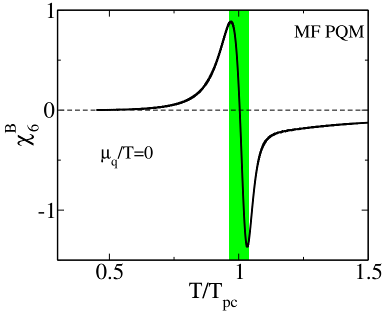
$U(\sigma, \boldsymbol{\pi}) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - c\sigma$ is meson potential

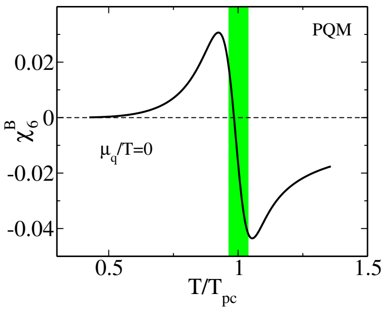
Will be also discussed in M. Nahrgang's talk

- **accounts for universal critical behaviour near chiral transition**
- reproduces scaling properties and critical exponents
- respects symmetries (Goldstone theorem fulfilled, second-order phase transition for $O(N)$ model)

Temperature dependence of χ_6^B in chiral models

Physical pion mass

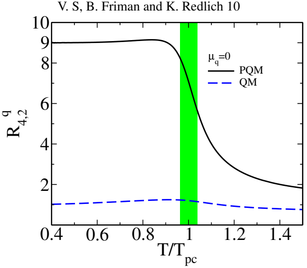




Green region shows crossover transition defined by the chiral susceptebility

What is responsible for this behavior?

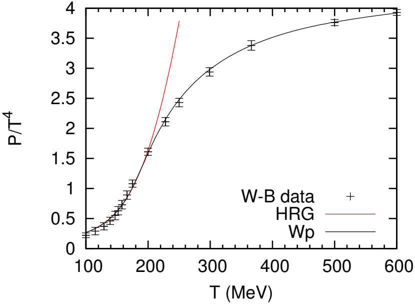
Change of the degrees of freedom:



- $R^q_{4,2} = \frac{1}{9} R^B_{4,2} = \chi^q_4 / \chi^q_2$ quark content of effective degrees of freedom that carry baryon number
- $R^q_{4,2}(T \to 0) \to 9 \rightsquigarrow$ three quark degrees of freedom carrying baryon charge
- $R^q_{4,2}(T \gg T_{pc}) \to 6/\pi^2 \approx 1 \rightsquigarrow$ single-quark dof carrying baryon charge

S. Ejiri, F. Karsch and K. Redlich 05

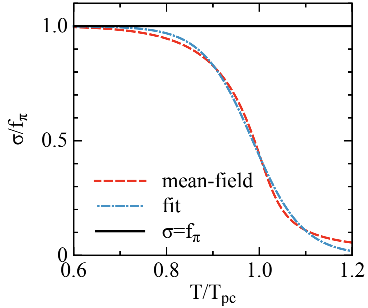
Borsanyi et al., 2010:



- Change of the degrees of freedom (deconfinement) may also alter higher order cumulants
- In a model one can freeze chiral dynamics. (In LQCD one might consider quarks in adjoint representation to separate two transitions apart)

What is responsible for this behavior?

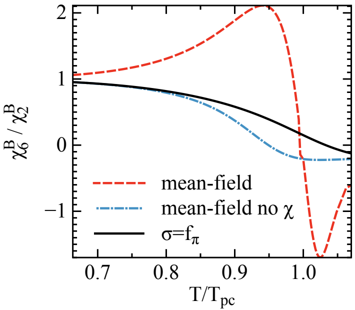
Chiral condensate:



Three cases

- Mean-field chiral model with dynamical σ field
- Frozen $\sigma = f_\pi$
- With σ given by a function fitted to reproduce $\sigma(T)$

$R^B_{6,2} = \chi^B_6 / \chi^B_2$:



- Criticality related to **chiral condensate is responsible** for non-trivial structure in $R^B_{6,2}$

Volume fluctuations

Volume fluctuations is one of effects that may change cumulants. Higher order cumulants are most sensitive.

Assumptions

- volume fluctuations are not critical (independent of baryon fluctuations)

$$P(V,B) = \mathcal{P}(V)P(B)$$

- other thermodynamic quantities do not fluctuate

The last assumption can be justified at high energies ($\mu_B \to 0$).
If system thermalizes, the finite temperature is independent of initial one.

- Cumulant generating functions (CGF) for both fluctuations

$$\mathrm{CGF}^B(t) = \ln \sum_{B=-\infty}^{\infty} P(B) \exp(B\,t)$$

$$\mathrm{CGF}^V(s) = \ln \int_0^{\infty} dV\, \mathcal{P}(V) \exp(Vs)$$

- The additivity of cumulants and thermodynamic principles imply that

$$\mathrm{CGF}^B(t) = V \cdot \zeta^B(t),$$

where ζ^B is a volume-independent function.

Volume fluctuations

Aim is to obtain cumulants with volume fluctuations. These cumulants are obtained from the cumulant generating function

$$\phi^B(t) = \ln \int dV\, \mathcal{P}(V) \sum_B P(B) e^{Bt}$$

But

$$\sum_B P(B) e^{Bt} = e^{V\zeta^B(t)}$$

and consequently

$$\phi^B(t) = \ln \int dV \mathcal{P}(V) e^{V\zeta^B(t)}$$

From comparison with definition of cumulant generating function of volume fluctuations:

$$\phi^B(t) = \mathrm{CGF}^V\left(\zeta^B(t)\right)$$

Corresponding reduced cumulants are given by Taylor expansion of about $t = 0$

$$c_n = \frac{1}{\langle V \rangle} \frac{d^n}{dt^n} \phi^B(t)\Big|_{t=0}$$

Let κ_n are the cumulants for baryon fluctuations and v_n are for volume fluctuations ($\delta X = X - \langle X \rangle$)

$$\kappa_1 = \frac{1}{V}\langle B \rangle, \quad \kappa_2 = \frac{1}{V}\langle (\delta B)^2 \rangle, \quad \kappa_4 = \frac{1}{V}\left[\langle (\delta B)^4 \rangle - 3\langle (\delta B)^2 \rangle^2\right]$$

$$v_1 = \frac{1}{V}\langle V \rangle_V, \quad v_2 = \frac{1}{V}\langle (\delta V)^2 \rangle, \quad v_4 = \frac{1}{V}\left[\langle (\delta V)^4 \rangle - 3\langle (\delta V)^2 \rangle^2\right]$$

Then wanted cumulants of baryon number fluctuations including volume fluctuations are given by

$$\begin{aligned} c_1 &= \kappa_1 \\ c_2 &= \kappa_2 + {\kappa_1}^2 v_2 \\ c_3 &= \kappa_3 + 3\kappa_2\kappa_1 v_2 + {\kappa_1}^3 v_3 \\ c_4 &= \kappa_4 + (4\kappa_3\kappa_1 + 3{\kappa_2}^2)v_2 + 6\kappa_2{\kappa_1}^2 v_3 + {\kappa_1}^4 v_4 \\ \ldots \end{aligned}$$

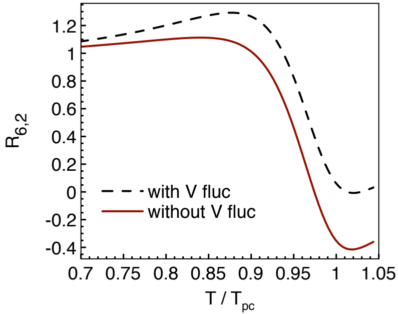
- At zero chemical potential $\kappa_{2n+1} \to 0$.
- It is also useful to work in terms of baryon number susceptibilities $\chi_n^B = \kappa_n / T^3$ and dimensionless cumulants of volume fluctuations V_n appropriately scaled by mean volume $\bar{V}$, e.g. $V_2 = \frac{1}{\bar{V}^2}\langle(\delta V)^2\rangle$.
- Ratios of cumulants:

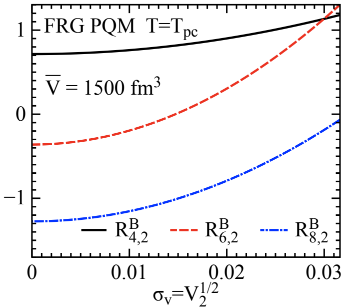
$$R_{4,2}^{B\&V} \equiv c_4/c_2 = R_{4,2}^B + 3(\bar{V}T^3)V_2\chi_2^B; \quad R_{4,2}^B = \chi_4^B/\chi_2^B$$

$$R_{6,2}^{B\&V} \equiv c_6/c_2 = R_{6,2}^B + 15(\bar{V}T^3)V_2\chi_4^B + 15(\bar{V}T^3)^2V_3(\chi_2^B)^2$$

  - volume dependence does not cancel out
  - if volume probability distribution has negative skewness, $V_3 < 0$, it may change sign of $R_{6,2}^{B\&V}$.

Assume symmetric probability distribution for volume fluctuations ($V_{2n+1} = 0$)





- Non-monotonic dependence on order of cumulants
- Even small 2% fluctuations of volume around mean value strongly modify ratios
- Under assumption $V_{2n+1} = 0$, result for $R_{4,2}$ and $R_{6,2}$ are independent on higher order cumulants of volume fluctuations.

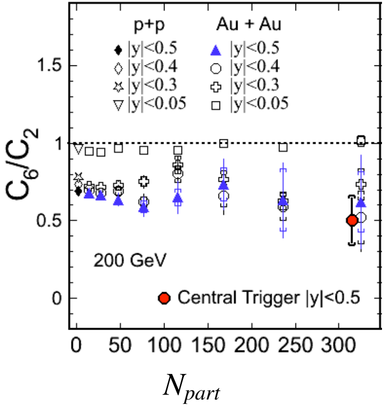
Ratios depend on mean volume $\bar{V}$

$$R^{B\&V}_{6,2} \equiv c_6/c_2 = R^B_{6,2} + 15(\bar{V}T^3)V_2\chi^B_4 + 15(\bar{V}T^3)^2 V_3(\chi^B_2)^2$$

STAR result on proton $R_{6,2} = c_6/c_2$

- in first approximation, mean volume is proportional to N_{part}
- experimental c_6/c_2 shows no volume dependence; this means either $(R^{B\&V}_{6,2} - R^B_{6,2})$ is small, or there is cancellation with other effects (e.g. baryon conservation)

STAR preliminary (L. Chen, QM2012)



Conclusions

- Properties of QCD chiral crossover can be studied by analysis of sixth order cumulant of baryon number fluctuations (the same is true for electric charge fluctuations)
- If freeze-out happens close to the transition, sixth order cumulant would be negative or smaller then corresponding HRG values
- Volume fluctuations at zero chemical potential tend to increase ratios of cumulants and may drive c_6/c_2 from negative to positive values (in contrast to baryon conservation, which suppresses ratios)
- Experimentally measured c_6/c_2 shows little dependence on N_{part}. Does this mean that volume fluctuations are negligible? Or there is non-trivial cancellation of different effects?