Hadrons and the phase diagram of QCD

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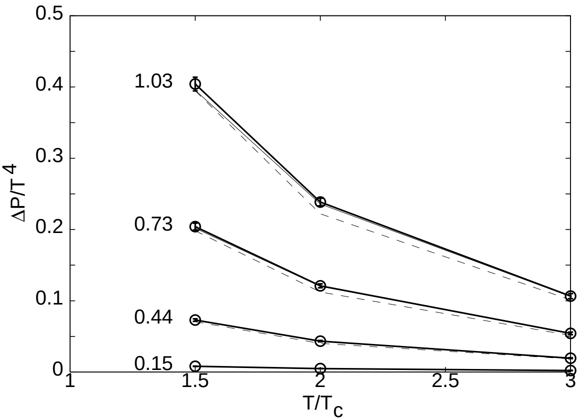
EOS at $\mu \neq 0$



Gavai, SG: Phys.Rev. D68 (2003) 034506

$$\Delta P = P(T, \mu) - P(T, 0).$$

The mathematical problem

Given the series expansion of a function in powers of z

$$f(z) = \sum_{i=0}^{\infty} f_{2i}(0) \frac{z^{2i}}{(2i)!},$$

at $z = 0$, reconstruct the function. Well studied classical problem!

Special complications: only a small number of series coefficients are known, the coefficients are known within errors.

Simplest part of the problem is to estimate whether the series is summable. Usual notions of radius of convergence. Next more complicated, estimating the value of the function for $z \simeq \sqrt{f_0/f_2}$. Both questions addressed here.

The simulations

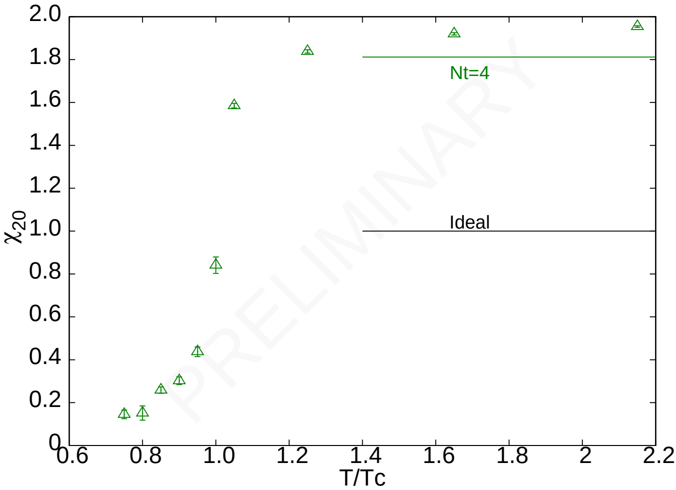
Lattice simulations with $N_t = 4$, 6 and 8. Fixed $m_\pi \simeq 220$ MeV. Spatial volume of around 100 fm^3.

First results presented in QM 2012: at each coupling 100 statistically independent gauge configurations and 1000 noise vectors per configuration. Now updated with 200 statistically independent gauge configurations at each coupling and 2000 noise vectors per coupling.

More statistics on disk. Present results are a progress report.

Datta, Gavai, SG: arXiv:1210.6784

Lattice spacing effects



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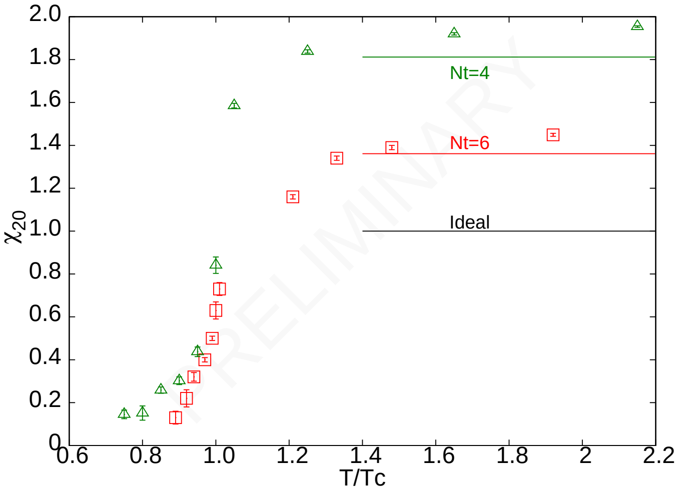

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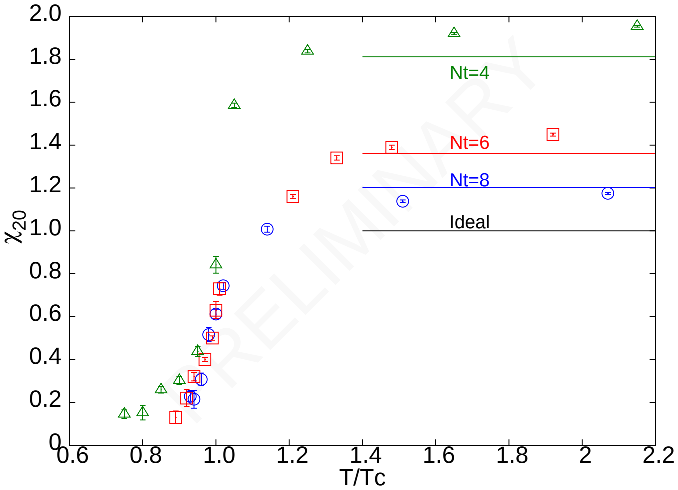
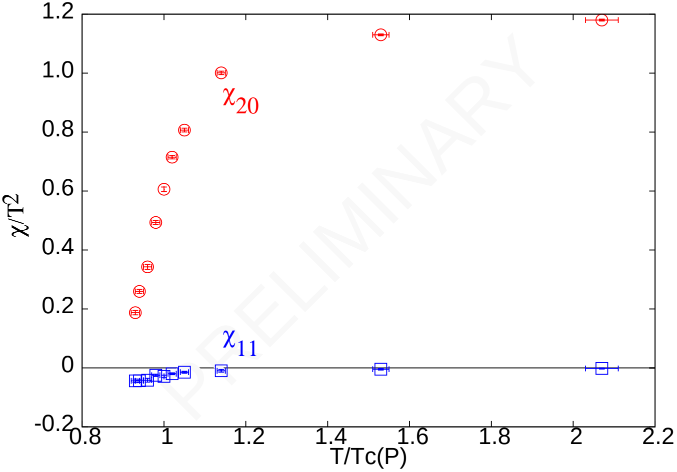


Susceptibilities at $\mu = 0$



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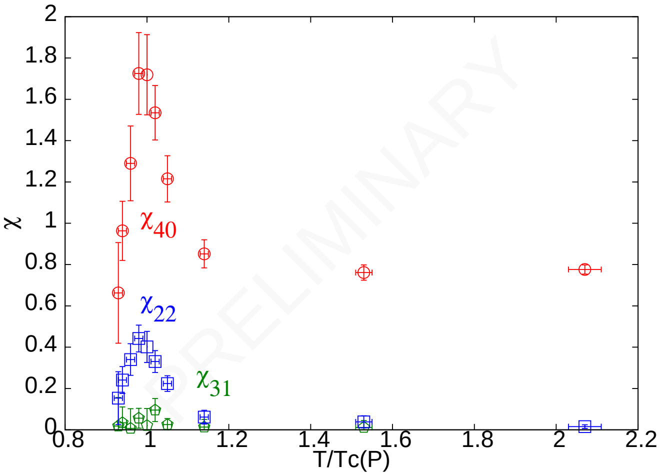


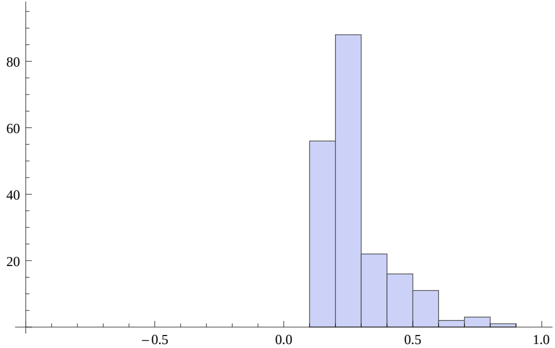
Series expansion

Series expansion of pressure and its derivatives in powers of $z = \mu/T$ at fixed T. Successive terms of the series increase for $T < T_c(P)$. Implies a finite radius of convergence of series.

Radius of convergence bounded by a singularity somewhere in the complex plane. Corresponds to a critical point if singularity on the real axis.

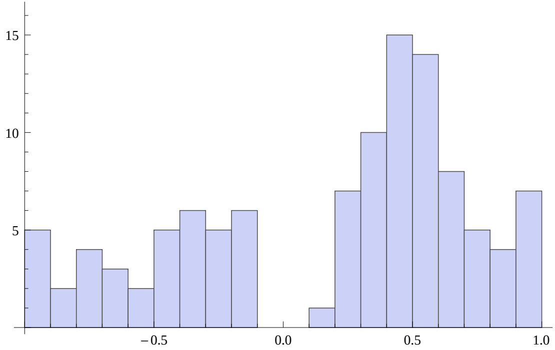
In numerical series expansion to finite order, important to test this criterion for each definition of the radius of convergence.

Bootstrap statistics



Bootstrap histogram of $z_*^2 = \chi_2^{(B)}/\chi_4^{(B)}$.

Bootstrap statistics



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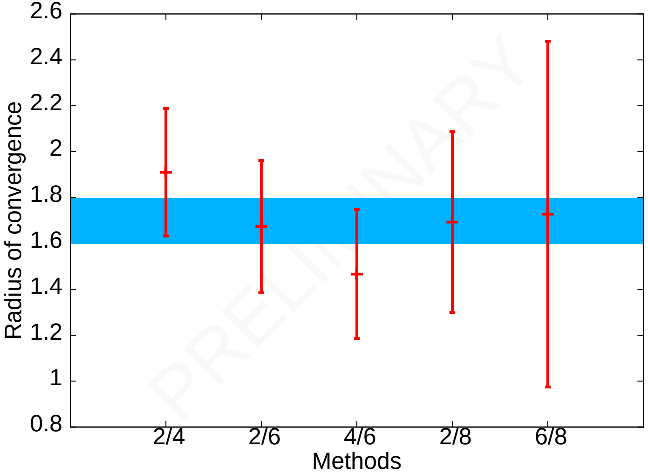
The radius of convergence



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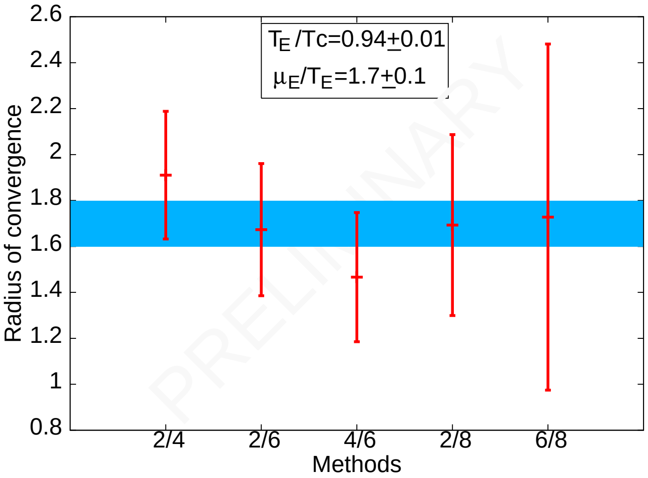


Susceptibility for $\mu \neq 0$

Resum the series into a Padé approximant. For example,

$$[0,1]: \qquad \frac{\chi_{20}(z)}{T^2} = \frac{c}{z_* - z}$$

$$[1,1]: \qquad \frac{\chi_{20}(z)}{T^2} = \frac{a + bz}{z_* - z}$$

Width of the critical region? If we define it by

$$\left|\frac{\chi_{20}(z)}{\chi_{20}(0)}\right| > \Lambda,$$

then for [0, 1] Padé: $|z - z_*| \leq z_*/\Lambda$.
Errors in extrapolation? Simple case of [0, 1] Padé, we have

$$\left|\frac{\Delta\chi_{20}}{\chi_{20}}\right| > \frac{1}{1 - \Lambda\delta},$$

where δ is fractional error in z_*. General case similar.

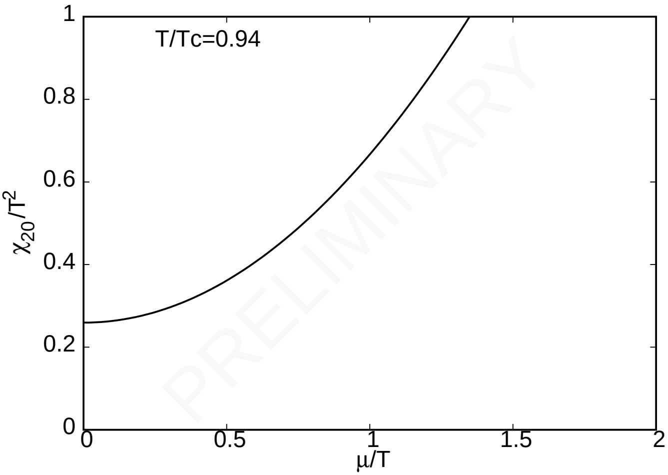
TANSTAAFL

Critical point has large fluctuations

Near a critical point in old-fashioned Monte Carlo simulations: critical slowing down. In numerical series expansions: large statistical errors.

Large statistics needed in order to measure physical quantities near a critical point. In usual Monte Carlo simulations: large autocorrelation lengths signal approach to a critical point. In numerical series expansions: strong statistics dependence of extrapolations.

Susceptibility for $\mu \neq 0$



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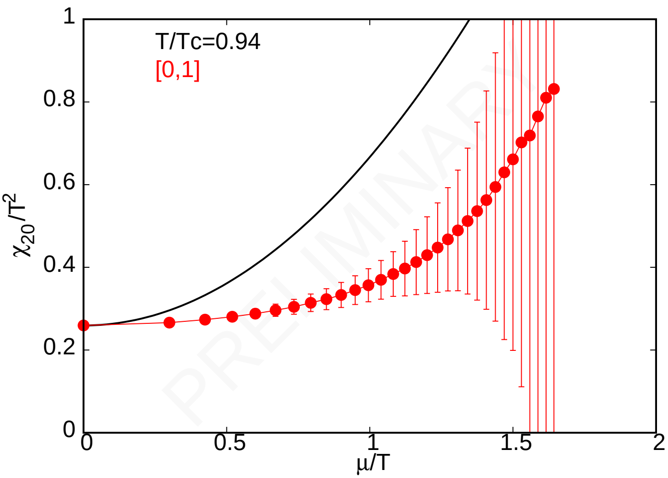

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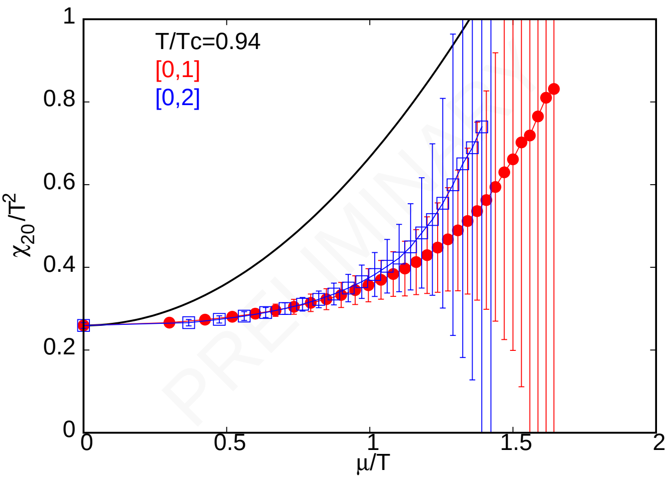

A divergence

Want to evaluate the $[0, 1]$ Padé approximant

$$P(z; a) = \frac{1}{z - a},$$

at various $z = \mu_B/T$ for a determined from lattice measurement. If a has Gaussian errors, then for any z, there is a probability that $a = z$. So the mean and variance of P both diverge.

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See this another way. Assume that the distribution of a is Gaussian with mean 1 and variance σ^2. Then the distribution of P at fixed z is given by

$$p(P;z) = \frac{1}{\sqrt{2\pi\sigma^2}}\,\frac{1}{P^2}\,\mathrm{e}^{-(z-1-1/P)^2/(2\sigma^2)}.$$

The distribution is normalizable but none of the moments exist.

The regularization

Yes. Because of finite statistics the maximum and minimum values of the Padé approximant are always bounded.

If one estimates $P(z;a)$ by a bootstrap, then one should take the number of bootstrap samples to be $\mathcal{O}(N)$. By accounting for the restricted range $|P|\le\Lambda$, all the integrals are regularized. If the measurements are made with statistics of N, then $\sigma^2\propto 1/N$. In generic samples

$$\epsilon(\Lambda)=1-\int_{-\Lambda}^{\Lambda}dPp(P;z),$$

where Λ is such that $N\epsilon(\Lambda)\ll 1$.

With $\sigma^2\propto 1/N$ and $\epsilon\propto 1/N$, in the limit $N\to\infty$ is it possible to remove the regularization and have finite $\langle P\rangle$ and $\langle P^2\rangle$?

Finite results: renormalization

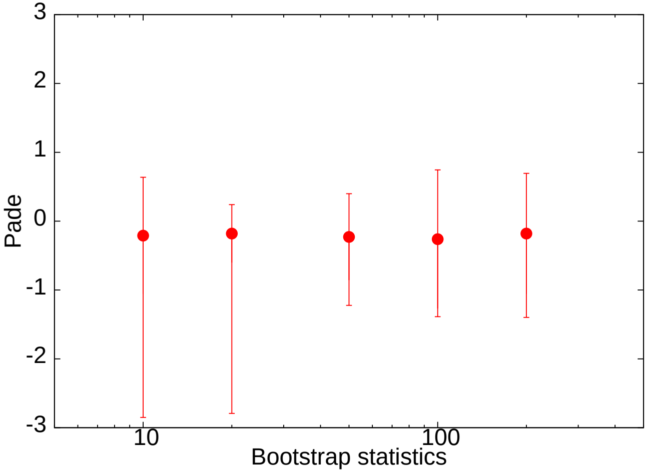
With increasing N one can arrange $N\epsilon$ to be constant by scaling $\Lambda \to \zeta\Lambda$ with $\zeta \propto N^{3/2}$. For Gaussian distributed a,

$$\delta\langle P\rangle \simeq \mathrm{e}^{-K(1-z)^2 N} \log(\zeta/\zeta')$$

$$\delta\langle P^2\rangle \simeq \mathrm{e}^{-K(1-z)^2 N} (\zeta - \zeta')\Lambda\sigma$$

As a result a bootstrap estimation will lead to good estimates of mean and error except for $|z-1| < \mathcal{O}(1/\sqrt{N})$.
Beyond the Gaussian approximation: bound the growth of $\langle P\rangle$ and $\langle P^2\rangle$ by verifying that the estimate of the error in the pole narrows faster than the growth of the probability in the tail of the distribution of the value of $P(z;a)$.
Numerical experiments work when a is the ratio of two Gaussian distributed variates (each with variance going as $1/N$).

The theory works



Ready to give ΔP close to the critical point.

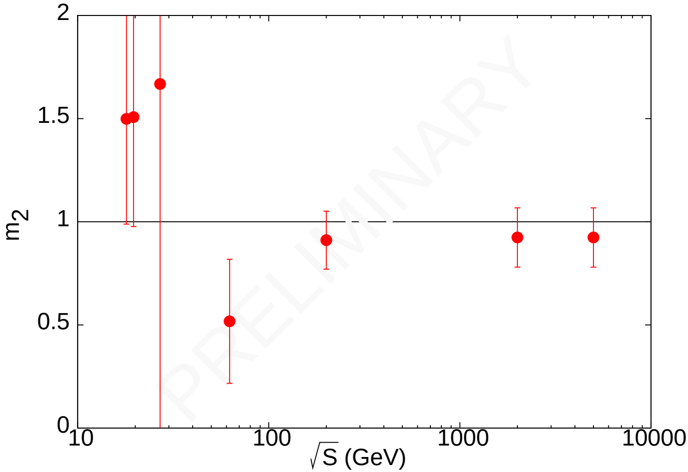
Fluctuations

Extrapolate ratios of cumulants to freezeout curve; make contact with experimental measurements. **Gavai, SG: Phys.Rev. D83 (2011) 074510**

Caveats noted:

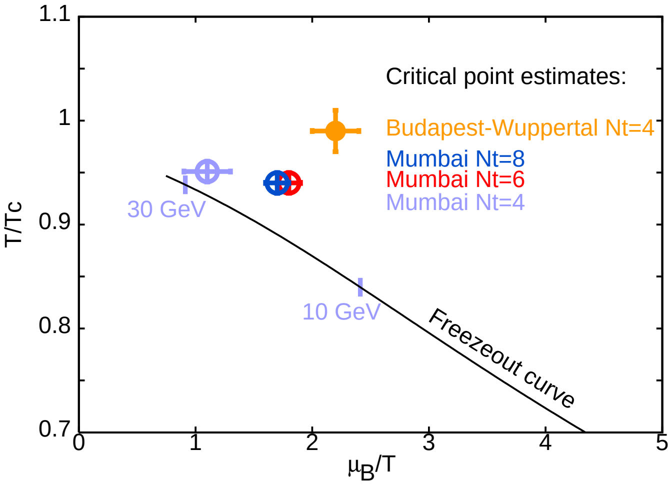
1. Time scale for reaction $p \leftrightarrow n$ to be included.
   Asakawa, Kitazawa: Phys.Rev. C85 (2012) 021901
2. Observation volume, V_{obs}, selected by detector. Comparison to lattice works when $\xi^3 \ll V_{obs} \ll V_{fireball}$.
3. Scale of the persistence of memory, $V_{fireball}$. When $V_{fireball}/V_{obs} \gg 1$ then overall conservation forgotten.
   Bzdak, Koch, Skokov: Phys.Rev. C87 (2013) 014901
4. Shortest length scale ξ, controls scale at which diffusion of B becomes important. **Datta et al: JHEP 1302 (2013) 145**
5. Dynamics needed: Peclet scale, $\lambda = \xi/M$ (M is the Mach number). **Bhalerao, SG: Phys.Rev. C79 (2009) 064901**

Fluctuations



SG: Nucl.Phys. A830 (2009) 749C; Athanasiou et al: Phys.Rev. D82 (2010) 074008

Critical point and critical region



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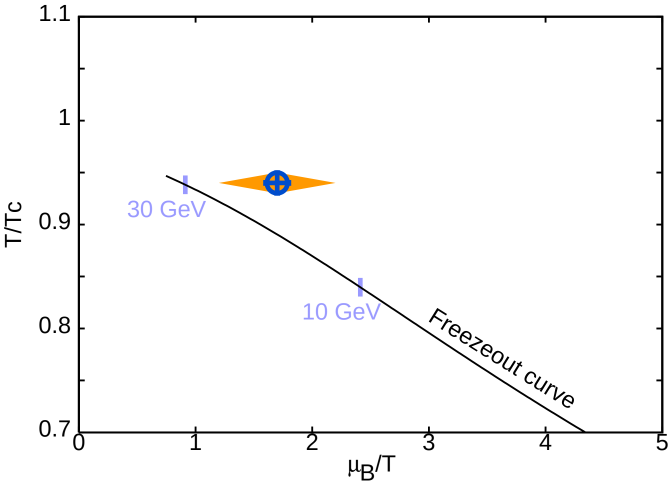

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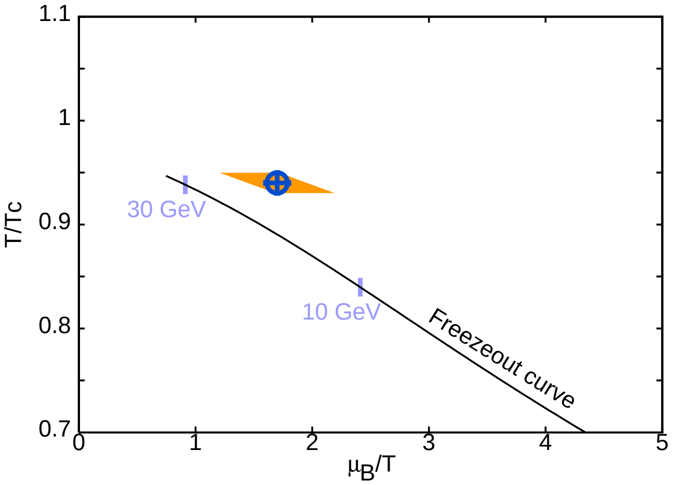

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