QCD phase diagram, fluctuations and the critical point

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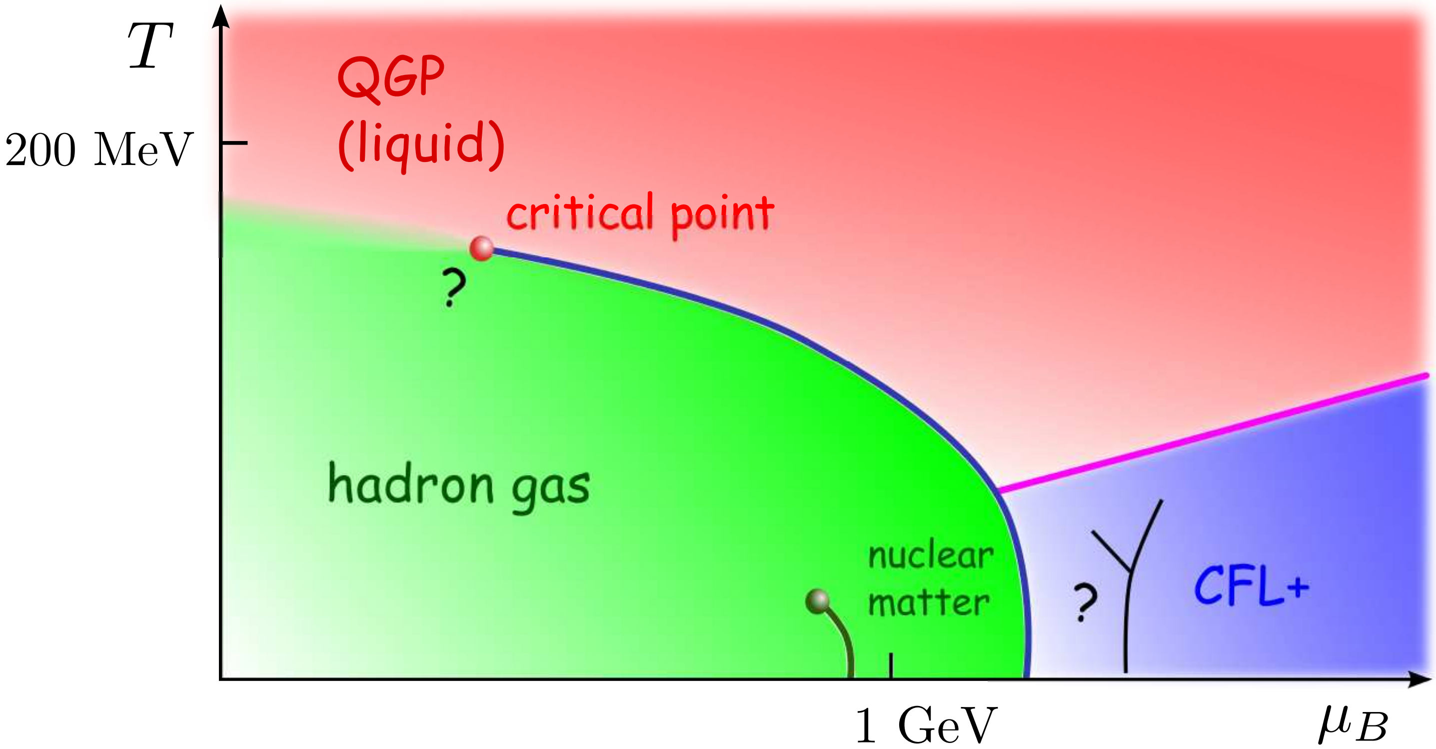
QCD Phase Diagram



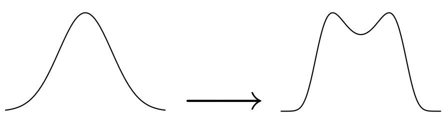
Critical point and fluctuations

The key equation:

$$P(x) \sim e^{S(x)} \qquad \textit{(Einstein 1910)}$$

At the critical point $S(x)$ has a “flat direction” or “soft-mode”. Fluctuations diverge:

$$\langle\, x^2 \,\rangle = -\left(\frac{\partial^2 S}{\partial x^2}\right)^{-1} = T\chi.$$



In grand canonical ensemble $P \sim e^{S-E/T-\mu N/T} = e^{pV/T}$

Fluctuations of order parameter and ξ

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots\right]. \qquad \Rightarrow \qquad \xi = m_\sigma^{-1}$$

- Moments (connected) of $\boldsymbol{q} = 0$ mode $\sigma_V \equiv \int d^3x\, \sigma(x)$:

$$\langle\, \sigma_V^2 \,\rangle = VT\,\xi^2\,; \qquad \langle\, \sigma_V^3 \,\rangle = 2VT^2\,\lambda_3\,\xi^6\,;$$

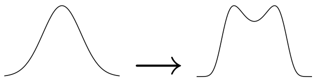
$$\langle\, \sigma_V^4 \,\rangle_c \equiv \langle\, \sigma_V^4 \,\rangle - 3\,\langle\, \sigma_V^2 \,\rangle^2 = 6VT^3\,[\,2(\lambda_3\xi)^2 - \lambda_4\,]\,\xi^8\,.$$

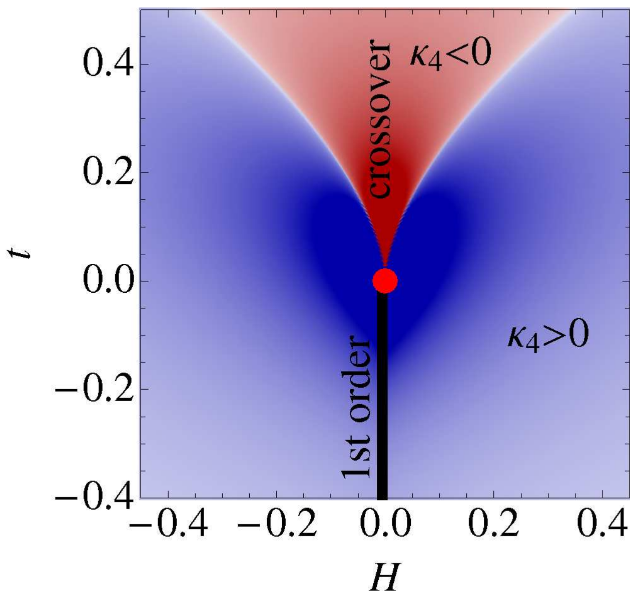
- Tree graphs. Each propagator gives ξ^2.

- Scaling requires “running”: $\lambda_3 = \tilde\lambda_3 T(T\xi)^{-3/2}$ and $\lambda_4 = \tilde\lambda_4 (T\xi)^{-1}$, i.e.,

$$\langle\, \sigma_V^3 \,\rangle = 2VT^{3/2}\,\tilde\lambda_3\,\xi^{4.5}\,; \quad 6VT^2\,[\,2(\tilde\lambda_3)^2 - \tilde\lambda_4\,]\,\xi^7\,.$$

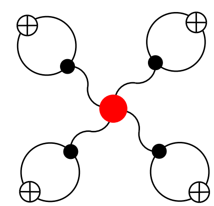
Negative kurtosis

- Not only kurtosis becomes large, but it also changes sign (PRL 107:052301,2011)
- $P(\sigma_V)$:  Thus $\langle\, \sigma_V^4 \,\rangle_c < 0$ on the crossover line ($\lambda_3 = 0$). And around it.



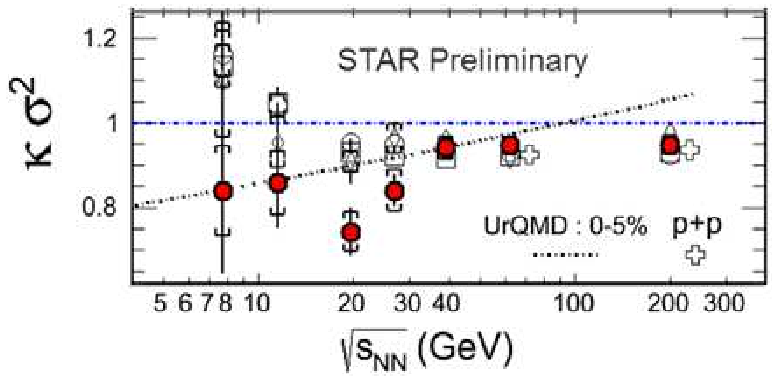
- Universal Ising eq. of state $M(H)$:
  $M = R^\beta \theta, \qquad t = R(1-\theta^2), \qquad H = R^{\beta\delta} h(\theta)$
  - here κ_4 is $\kappa_4(M) \equiv \langle\, M^4 \,\rangle_c$
  - in QCD $M \to \sigma_V$, and $(t, H) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$

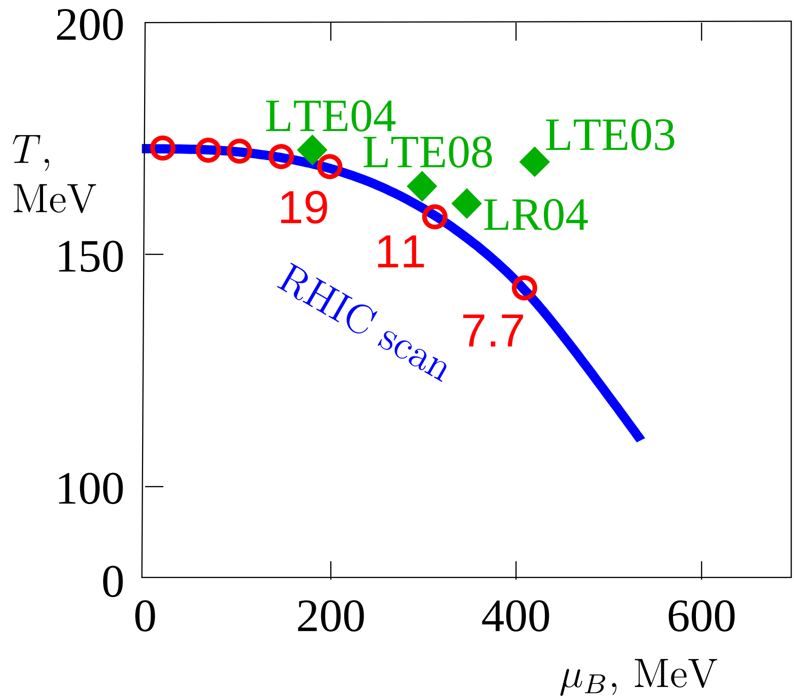
$$\langle\, (\delta N)^4 \,\rangle_c = \langle\, N \,\rangle + \langle\, \sigma_V^4 \,\rangle_c \left(\frac{g}{T} \int_{\boldsymbol p} \frac{v_{\boldsymbol p}^2}{\gamma_{\boldsymbol p}} \right)^4 + \ldots ,$$



$\langle\, \sigma_V^4 \,\rangle_c < 0$ means $\quad \omega_4(N) \equiv \langle\, (\delta N)^4 \,\rangle_c / \langle\, N \,\rangle < 1$

RHIC energy scan



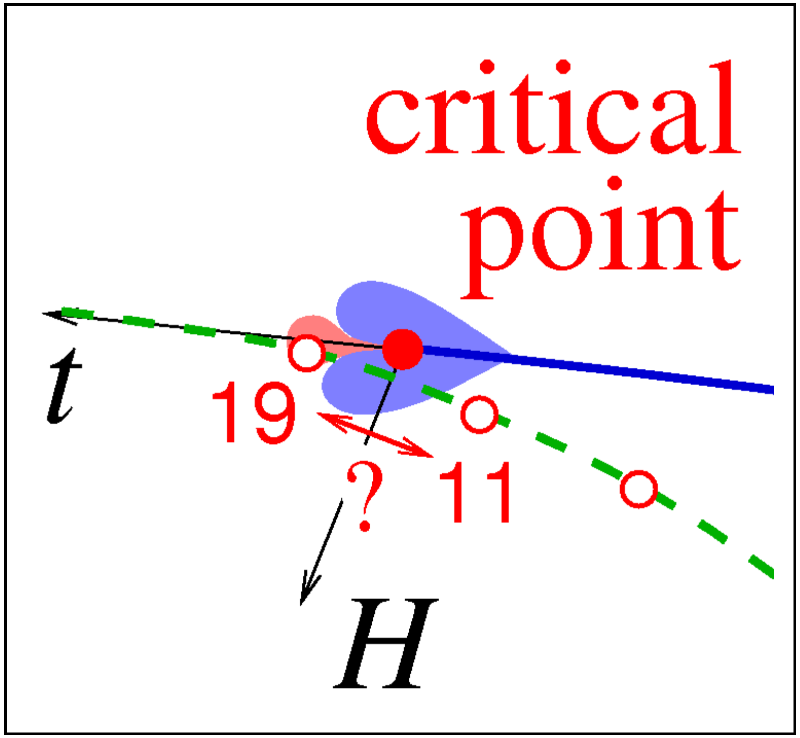


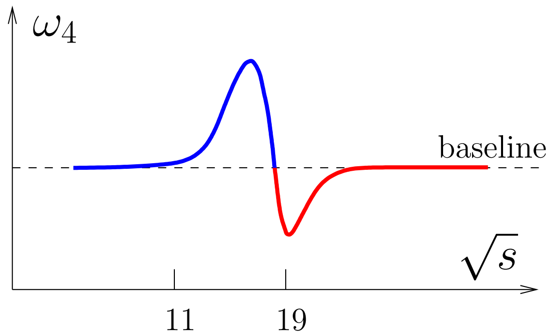
- 3 points at $\mu_B > 200$ MeV.

- O(magnitude) agreement with Athanasiou-Rajagopal-MS at 19 GeV.

  Effects of acceptance are expected to be large (A. Bzdak's talk).

A scenario





- Critical region $\Delta\mu_B \sim \mathcal{O}(100-150)$ MeV.

Fluctuations and Noise

Einstein (1910):

$$P(x) \sim e^{S(x)}$$

- Onsager:

$$\dot{x} = \gamma \frac{\partial S}{\partial x} + y$$

  with $\langle\, y(t_1) y(t_2) \,\rangle = 2\gamma\delta(t_1 - t_2)$ where γ — Onsager coeff.

- In relativistic Landau-Lifshits hydrodynamics *(Kapusta, Müller, M.S.)*

$$\nabla_\mu \left(T^{\mu\nu}_{\text{ideal}} + \Delta T^{\mu\nu} + S^{\mu\nu} \right) = 0 \qquad \text{and} \qquad \nabla_\mu \left(n u^\mu + \Delta J^\mu + I^\mu \right) = 0$$

  $\gamma = \lambda T \boldsymbol{q}^2$, where $\lambda = \sigma, \eta, \zeta$.

- The equlibrium amplitude is $\langle\, x^2 \,\rangle = -\left(\frac{\partial^2 S}{\partial x^2} \right)^{-1} = T\chi$.

  At the critical point it diverges due to the “soft mode”.
  (See this in M. Nahrgang’s talk).

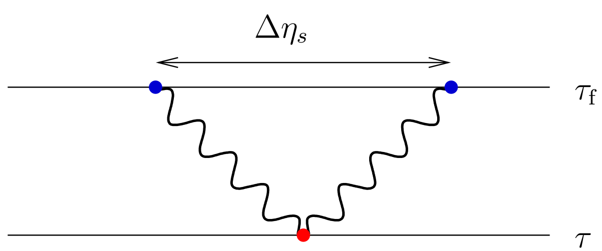
- Static equilibrium:
  (i) only χ matters, not λ; (ii) equal-time correlations are local.

Hydrodynamic correlations from local noise

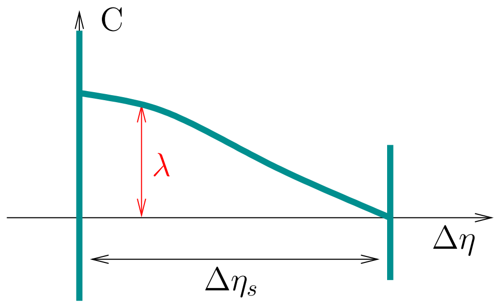
In Bjorken coordinates: $ds^2 = -d\tau^2 + \tau^2 d\eta^2 + d\boldsymbol{x}_\perp^2$

- Sound propagation: $\dfrac{dl}{dt} = v_s$ — sound speed in L.R.F.

$$\left.\begin{aligned} dl &= \tau d\eta \\ dt &= d\tau \end{aligned}\right\} \quad \Rightarrow \quad \frac{\tau d\eta}{d\tau} = \frac{d\eta}{d\log\tau} = v_s \text{ and } \Delta\eta_s = 2\int_\tau^{\tau_f} v_s \frac{d\tau}{\tau}$$



- Wake:



At the critical point kinetic coefficients also diverge.

Kapusta, Torres-Rincon consider $\sigma \sim \xi^{1\cdot}$.

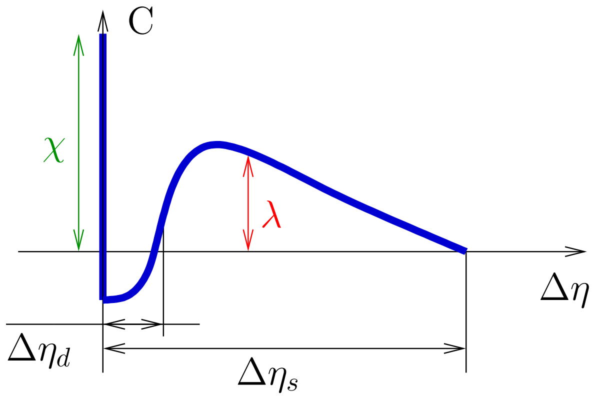
It would be interesting to see effect of $\zeta \sim \xi^{3\cdot}$.

Diffusion and correlations

In L.R.F.: $dl^2 = Ddt \;\; \Rightarrow \;\; \tau^2 d\eta^2 = Dd\tau$.

$$(\Delta\eta_d)^2 = \int_\tau^{\tau_f} D\frac{d\tau}{\tau^2}$$

(finite as $\tau_f \to \infty$). $D = \lambda/\chi$.



$\Delta\eta_d \sim 1 - 1.5$ (balance function width)

$\Delta\eta_s \sim 3 - 4$ (sound horizon)

Work in progress *(B. Ling, T. Springer, M.S.)*

Summary/Outlook

- Critical point is a special singular point on the phase diagram, with unique signatures. This makes its experimental discovery possible.
- Higher moments of fluctuations are sensitive signatures of the critical point. The effects of conservation laws, finite acceptance, and time evolution need to be studied.
- The effect of the expansion on the fluctuations and correlations can be studied using hydrodynamics with noise.
- The experimental search for the critical point is on.
  More measurements at $\sqrt{s}$ values below 19 GeV are needed to map QCD phase diagram.